Optimizing Multi-copy Two-hop Routing in Mobile Social Networks

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***Abstract*—In this paper, an opportunistic multi-copy two-hop routing algorithm is proposed for mobile social networks (MSNs) to minimize the expected data delivery delay, using local information. For each source-destination pair, the source dynamically maintains a *forwarding set* consisting of relay nodes. The forwarding set selection is based on the number of remaining message copies, as well as the number and quality of relays that have not received a message copy. The source only forwards its message to the relay nodes in its forwarding set, which will in turn forward the message to the destination directly. We propose a greedy approach to select the forwarding set with n message copies at the source, in an MSN with m (m>n) relays. All forwarding sets can be determined with a time complexity of $O(m \log m+nm)$. Then, the proposed multi-copy two-hop routing algorithm is applied to a feature space routing scheme, where the contact frequencies are estimated by social feature distances. Finally, the competitive performance of the proposed schemes are shown in real trace-driven simulations.**

Keywords*—Mobile social networks, multi-copy routing, opportunistic routing, social features, two-hop routing.

I. Introduction

Mobile social networks (MSNs), a new type of delay tolerant networks [1], are designed to operate without the support of preset infrastructures and guaranteed network connectivity. In MSNs, nodes use opportunistic contacts for communications while coping with such intermittent connectivity, which enables message delivery even if end-to-end paths never exist. Due to limited network information, two-hop routing has been proposed [2–4], which uses *local network information* (i.e., neighbors and neighbors' neighbors). Moreover, [3] shows that two-hop routing achieves a high delivery ratio through mobility. In two-hop routing, each node records its two-hop neighbor information to obtain source-relay-destination paths (denoted as S-R-D, in Fig. 1). Then, the source selects relays for message delivery, and relays are restricted to forwarding the message only to the destination. Therefore, each message copy will be forwarded at most twice, resulting in the advantage of the bounded resource (e.g., energy and buffer) consumption.

However, it still remains a major challenge to minimize data delivery delay of the two-hop routing, given limited message copies (or simply copies) at the source. The copy limitation is in consideration of the resource consumption: more copies brings a smaller delay, as well as a higher resource consumption. A two-hop routing example is shown in Fig. 1 with m=3 paths. The number on each link indicates the average delay of this link. Then, let us use one message copy (i.e., n=1) to illustrate the derivation of the *minimum expected delay*

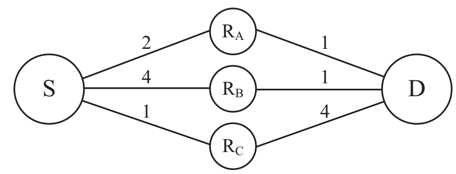


Fig. 1. An illustration of a two-hop routing model (source-relay-destination). The number on each link indicates the average delay of this link.

TABLE I. Forwarding Set Outlook

Forwarding set of the 1^{st} copy (when the source has 2 copies)	Forwarding set of the 2^{nd} copy (when the source has 1 copy)
$\{R_A, R_B, R_C\}$	R_A gets the 1^{st} copy $\Rightarrow \{R_B, R_C\}$
	R_B gets the 1^{st} copy $\Rightarrow \{R_A\}$
	R_C gets the 1^{st} copy $\Rightarrow \{R_A, R_B\}$

(MED) of the two-hop routing. If the source always forwards the message to the first encountered node (mostly likely R_C in Fig. 1), the performance is poor, since $delay(R_C\text{-}D)$ is large. A better strategy is to choose the best-path (S-R_A-D) with delay 2+1=3. In this case, the message is only forwarded to R_A, regardless of contacts with R_B and R_C. However, the source should also forward the message to R_B, if it meets R_B before R_A. This is because $delay(R_B\text{-}D)<delay(S\text{-}R_A\text{-}D)$. Therefore, opportunistic routing [5] is introduced to enhance the two-hop routing. It means that the source is aware of a set of potential forwarders, called *forwarding set*. Then, the source only forwards its copy to encountered relays in its forwarding set $\{R_A, R_B\}$, ignoring R_C even if it is the next encounter.

In this paper, our objective is to the MED of the multi-copy two-hop routing. However, dynamically determining the forwarding set for the multi-copy two-hop routing is still a challenging problem. If we have more message copies, more relays should be selected in the forwarding set. In Fig. 1, if we have $n\geq3$ message copies, the source should give each relay a copy, and the forwarding set of the 1^{st} copy is $\{R_A, R_B, R_C\}$, rather than $\{R_A, R_B\}$ when the source has only one copy.

When the number of copies is less than 3, say n=2, the situation is more complicated. Obviously, R_A and R_B should be selected in the forwarding set of the 1^{st} copy. However, should R_C be selected in the forwarding set of the 1^{st} copy? Suppose the forwarding set of the 1^{st} copy is $\{R_A, R_B, R_C\}$, what is the forwarding set of the 2^{nd} copy? The final result is shown in Table I. If R_A gets the 1^{st} copy, the forwarding set of the 2^{nd} copy is $\{R_B, R_C\}$, since $delay(R_B\text{-}D)<delay(S\text{-}R_C\text{-}D)$ and $delay(R_C\text{-}D)<delay(S\text{-}R_B\text{-}D)$. However, if R_B gets the 1^{st} copy, the forwarding set of the 2^{nd} copy is $\{R_A\}$, due to $delay(R_A\text{-}D)<delay(S\text{-}R_C\text{-}D)$ and $delay(R_C$-

D)>$delay$(S-R_A-D). As we can see, the forwarding set of the 2^{nd} copy varies depending on the actual relay of the 1^{st} copy.

As shown in the above example, the forwarding set for the i^{th} copy, simply i^{th} forwarding set, may overlap with the j^{th} forwarding set ($j < i$). However, the actual relay for the j^{th} copy will be excluded from further consideration. The set excluding actual relays is called *residual relay set* and its elements are called *residual relays*. Note that the actual relay of the i^{th} copy is uncertain, since each relay in the i^{th} forwarding set has a probability to be the actual relay. So the forwarding set is dynamically decided, based on the number of remaining copies as well as the number and quality of residual relays. Moreover, if the forwarding set we selected for the current copy is too small, the subsequent copies will be blocked, losing the advantage of multiple copies. On the other hand, if the forwarding set we selected for the current copy is too large, this copy may end up choosing unqualified relays, i.e., this copy is useless. Therefore, this problem is very challenging.

Furthermore, we have two applications of the two-hop routing in this paper. The first one is the two-hop routing with the contact information, as described above. To relax the constraint that each copy in the two-hop routing can be forwarded at most twice, the two-hop routing is further applied to a multi-hop feature-based routing scheme, where we use the social feature information to estimate the contact information (people with more common features contact each other more frequently [6]). Then, this feature-based routing scheme decomposes its multi-hop routing process to a sequence of two-hop routings.

Preliminary studies on the single-copy two-hop routing scheme have been reported in [2–4], while the multi-copy case is much more challenging due to the dynamic nature of the residual relays. Meanwhile, the traditional feature-based MSN routing schemes [6–8] do not rely on the dynamic forwarding set, which captures the probabilistic nature of contacts. For example, [6] employs a routing strategy, where the message is forwarded to an inter-meeting relay if this relay has more common features with the destination than the message holder.

The main contribution of the paper is summarized as follows. (1) We propose a *performance-bounded* greedy approach for the multi-copy forwarding set selection. The key idea of the routing is to use local two-hop information to select an appropriate neighbor subset (i.e., forwarding set) to relay. All forwarding sets can be efficiently determined with a time complexity of $O(m \log m + nm)$, where m is the number of relays, and n is the number of copies at the source. (2) We analysis the *delivery delay reduction using multiple copies*, i.e., how additional copies can contribute to the delay reduction compared with the first copy. (3) To relax the constraint that each copy in the two-hop routing can be forwarded at most twice, the two-hop routing is further applied to a multi-hop feature-based routing scheme, which is decomposed to a sequence of two-hop routings.

The remainder of the paper is organized as follows: in Section II, we set up the model of two-hop routing, and explore the rules for the forwarding set selection; in Section III, the multi-copy two-hop routing algorithm is applied to a feature-based routing scheme for MSNs; in Section IV, the simulation results are shown; finally, in Section V, we conclude the paper. All proofs are presented in Appendix.

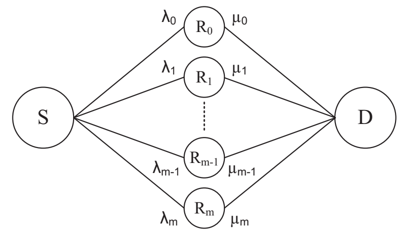


Fig. 2. Source-relay-destination paths (λ_i and μ_i indicate contact frequencies).

II. Multi-Copy Two-Hop Routing

In this section, we set up the two-hop routing model with the contact information. Then, a performance-bounded greedy approach is proposed for the forwarding set selection. Finally, we analyze the time complexity of the proposed algorithm.

A. System Model

First, we describe how nodes obtain the two-hop neighbor contact information. Each node in the network persistently records the link delay to its one-hop neighbors as the link-delay information through link probes, as described in [9]. According to [9, 10], link delays are generally modeled to follow *exponential distribution*. Meanwhile, nodes exchange their contact histories through two rounds of Hello messages with their neighbors to maintain two-hop information. In this section, we study a two-hop routing with the contact information, and in Section III, the two-hop routing is applied to a multi-hop feature-based routing scheme, where the contact information is estimated by the social feature information. Then, the two-hop graph (from source S to destination D) can be constructed as Fig. 2. There are m relay nodes $(R_1, R_2, ..., R_m)$ between S and D. The link delay between S and R_i follows exponential distribution with parameter λ_i, while the link delay between R_i and D follows exponential distribution with parameter μ_i. Note that parameters λ_i and μ_i indicate contact frequencies. In consideration of the direct contact between S and D, R_0 is attached for unified representation, where λ_0 is the corresponding parameter of link S-D and $\mu_0 = \infty$. In other words, the S-D path is transferred to be S-R_0-D (D becomes R_0-D). In addition, for presentation simplicity, the forwarding set $\{R_A, R_B\}$ is written as $\{A, B\}$ in the following equations.

B. Forwarding Set Selection

In the multi-copy two-hop routing, the source has n message copies initially. The data delivery is completed once a message copy reaches the destination, while the other message copies are deleted due to time-to-live. Our goal is the MED of the *multi-copy two-hop routing algorithm* (MTRA). Let F_n denote the forwarding set, and let E_n denote the corresponding expected delivery delay, when the source has n copies remaining. The PDF and CDF of the data delivery delay are denoted as $h_n(t)$ and $H_n(t)$, respectively. According to the definition of the forwarding set, the source with n remaining copies will forward a copy to an encountered relay in F_n, regardless of the other relays. After actual forwarding, the source remains $n{-}1$ copies, and a new forwarding set (i.e., F_{n-1}) is selected for that round. Therefore, $h_n(t)$ can be calculated as

$$h_n(t) = \int_0^t \sum_{i \in F_n} \{[\lambda_i e^{-\lambda_i T}][\prod_{j \neq i \& j \in F_n} e^{-\lambda_j T}][g^i_{n-1}(t-T)]\} \mathrm{d}T \qquad (1)$$

Here, $\lambda_i e^{-\lambda_i T}\mathrm{d}T$ presents the probability that the delay from S to R_i is in $[T, T+\mathrm{d}T]$. Then, the second term, $\prod e^{-\lambda_j T}$ (the subscript is omitted), shows the probability that R_i is the first relay in F_n that contacts S at time T. Let $g^i_{n-1}(t)$ denote the PDF of the smaller delay of (1) the delay of R_i-D path, and (2) the delay of data delivery with $n{-}1$ remaining copies for the residual relays. So the last term, $g^i_{n-1}(t{-}T)\mathrm{d}T$, shows the probability that the data delivery will be completed in $[t{-}T, t{-}T{+}\mathrm{d}T]$, if currently R_i has a copy and S has $n{-}1$ copies. Therefore, the corresponding CDF of $g^i_{n-1}(t)$ is

$$\begin{aligned} G^i_{n-1}(t) &= 1 - [1 - (1 - e^{-\mu_i t})][1 - H_{n-1}(t)] \\ &= 1 - e^{-\mu_i t}[1 - H_{n-1}(t)] \end{aligned} \quad (2)$$

For presentation simplicity, let us define

$$s_n = \sum_{i\in F_n} \lambda_i \quad \text{and} \quad s'_n = \sum_{i\in F_n} \lambda_i^2 \quad (3)$$

As shown in Appendix A, Eq. 1 is simplified to be

$$\begin{aligned} h_n(t) = &\sum_{i\in F_n} \frac{\lambda_i \mu_i}{s_n - \mu_i}[e^{-\mu_i t} - e^{-s_n t} - e^{-\mu_i t}H_{n-1}(t)] \\ &+ \sum_{i\in F_n} \frac{\lambda_i s_n e^{-s_n t}}{s_n - \mu_i} \int_0^t e^{(s_n-\mu_i)T} h_{n-1}(T)dT \end{aligned} \quad (4)$$

Then, the expected data delivery delay, E_n, can be calculated based on Eq. 4, as shown in Appendix B:

$$E_n = \frac{1}{s_n}[1 + \sum_{i\in F_n} \frac{\lambda_i}{\mu_i}] - \sum_{i\in F_n} \frac{\lambda_i}{s_n} \int_0^\infty e^{-\mu_i t} H_{n-1}(t)dt \quad (5)$$

When n=1, E_n is reduced to be the expected delivery delay of the single-copy two-hop routing, i.e., we have $H_{n-1}(t)$=0 and $E_1 {=} \frac{1}{s_1}[1{+}\sum_{i\in F_1} \frac{\lambda_i}{\mu_i}]$. In other words, the former part of Eq. 5 shows the expected delay of the first sent message copy, and the latter part in Eq. 5 indicates the decreased expected delay brought by the remaining $n-1$ copies. Moreover, if the forwarding set we selected for the current copy is too small, the subsequent copies will be blocked, losing the advantage of multiple copies. On the other hand, if the forwarding set we selected for the current copy is too large, this copy may end up choosing a unqualified relay, i.e., this copy is useless. This tradeoff is very challenging, and it is represented by the former and latter parts of of Eq. 5.

Now, let us explore the property of the optimal forwarding set (denoted by F_n^*) that minimizes E_n in Eq. 5 (the corresponding minimum E_n is denoted as E_n^*), and then we have

Theorem 1: If there are r ($r{\gg}n{\geq}1$) residual relays that have not received a copy, where $\mu_{k_1}{>}\mu_{k_2}{>}...{>}\mu_{k_r}$, then F_n^* satisfies $F_n^*{=}\{R_{k_1}, R_{k_2}, ..., R_{k_j}\}$ for a specified j in $[1, r]$.

The proof of Theorem 1 is shown in Appendix C, the insight meaning of which is shown as follows. For a relay node R_k, we can decide whether R_k is in the forwarding set or not, through comparing the delivery delay of passing a copy to R_k-D path and the delivery delay of not passing a copy. If the former delay is smaller, R_k should be included in the forwarding set (otherwise not). Since $r{\gg}n{\geq}1$, we can approximate the latter delay to a certain threshold. Therefore, if the delay of R_k-D path is smaller than this threshold, we should add R_k into the forwarding set, since waiting for the other relays takes more delivery time.

Although Theorem 1 shows the greediness property of the forwarding set selection, we cannot directly apply a greedy algorithm to minimize E_n. This is because E_n in Eq. 5 is so complex that we need exponential time to calculate it. Note that, in Eq. 5, $H_{n-1}(t)$ essentially varies with μ_i, since the residual relays are uncertain (the actual relay of the current copy is uncertain). Meanwhile, an accurate and simplified expression of E_n is too hard to obtain. Therefore, instead of minimizing E_n, we can minimize the *bound* of E_n. Since $0{\leq}H_{n-1}(t){\leq}1$, a naïve bound of E_n is

$$\frac{1}{s_n} \leq E_n \leq \frac{1}{s_n}[1 + \sum_{i\in F_n} \frac{\lambda_i}{\mu_i}] \quad (6)$$

The physical meaning of these bounds are that E_n should be larger than the expected delay from the source to one of the relays in F_n, while E_n should be smaller than the expected delay of the single-copy two-hop routing ($E_n \leq E_1$). Obviously, this bound is not tight, and a better bound is

Theorem 2: For the forwarding-set-based multi-copy two-hop routing algorithm, E_n satisfies

$$E_n \leq \frac{1}{s_n}[1 + \sum_{i\in F_n} \frac{\lambda_i \sqrt{E_{n-1}}}{\sqrt{2\mu_i}}] \quad (7)$$

The proof of Theorem 2 is shown in Appendix D. Theorem 2 shows the *performance bound* of our algorithm, which has insightful meanings as follows. Eq. 5 shows that the delay can be divided into two parts: the former part $\frac{1}{s_n}[1{+}\sum_{i\in F_n} \frac{\lambda_i}{\mu_i}]$ shows the delay of the currently sent copy; the latter part shows the delay reduction brought by the remaining $n{-}1$ copies. Meanwhile, the delay of the the currently sent copy, $\frac{1}{s_n}[1{+}\sum_{i\in F_n} \frac{\lambda_i}{\mu_i}]$, can also be divided into two parts: $\frac{1}{s_n}$ is the first hop delay; $\frac{1}{s_n}\sum_{i\in F_n} \frac{\lambda_i}{\mu_i}$ is the second hop delay. The key insight is that, the second hop delay of the currently sent copy should have *the same order of magnitude* with the delay reduction brought by the remaining $n{-}1$ copies: if the former one is the major delay, then we should select more qualified relays into the forwarding set of the current copy, i.e., remove unqualified relays; on the other hand, if the latter one is the major issue, then we should sent out the first copy as soon as possible to take full advantage of subsequent copies. Therefore, the delay reduction brought by the remaining $n{-}1$ copies can be approximated by the second hop delay of the currently sent copy, leading to the bound in Eq. 7. The tightness of this bound is sensitive to the relay qualities (i.e., λ_i and μ_i), which are complex for theoretical analysis. Therefore, the tightness test is done experimentally in Section IV.

Through recursion, Eq. 7 can be further derived to be

$$E_n < \frac{1}{s'_n}[\lambda_{max} + \frac{(2\mu_{max}E_1)^{\frac{1}{2^{n-1}}}}{2} \sum_{i\in F_n} \frac{\lambda_i^2}{\mu_i}] \quad (8)$$

The derivation process is shown in Appendix E. In Eq. 8, λ_{max} and μ_{max} are the maximum values of λ and μ for all initial paths (excluding $\mu_0 = \infty$ if $\lambda_0 > 0$). E_1 is the expected delay for the single-copy two-hop routing. Moreover, Eq. 8 shows the delivery delay reduction using multiple copies, i.e., how additional copies can contribute to the delay reduction compared with the first copy. The upper bound of the delivery delay includes (1) a constant part and (2) a part that decays

Algorithm 1 MTFSS

Input: The two-hop graph, copies number n;
Output: The forwarding set;

Sort residual relays by μ, $\mu_{k_1}>\mu_{k_2}>...\mu_{k_r}$;
Find λ_{max}, μ_{max}, and set F_1=F_n=$\varnothing$, E_1=EB_n=∞;
for $j = 1$ to r **do**
  if $\mu_{k_j} < 1/E_1$ **then**, **break**;
  Add relay R_{k_j} to F_1, and update E_1 by Eq. 10;
for $j = 1$ to r **do**
  if $\mu_{k_j} < C/EB_n$ and $n < j$ **then**, **break**;
  Add relay R_{k_j} to FB_n, and update EB_n by Eq. 9;
if $n = 1$ **then**
  return F_1;
else
  return FB_n;

Algorithm 2 MTRA-Source

Input: The two-hop graph, copies number n;

Add all relays to the residual relay set;
Call MTFSS to calculate F_n^*;
if R_k in F_n^* contacts S and $n > 0$ **then**
  S forwards one copy to R_k;
  Remove R_k from the residual relay set;
  $n = n - 1$ and call MTFSS to update F_n^*;

Algorithm 3 MTRA-Relay

Input: This relay holds a message copy or not;

if this relay holds a copy **then**
  if this relay contacts destination **then**
    Forward this copy to the destination;

exponentially with respect to the number of copies. In other words, the delay reduction contributed by an addition copy exponentially decays with respect to the number of copies.

Instead of minimizing E_n in Eq. 5 which is very complex, we can minimize the upper bound of E_n in Eq. 8. For presentation simplicity, let EB_n denote the upper bound of E_n (the right part in Eq. 8), with its minimum value EB_n^*. Let FB_n^* denote the optimal forwarding set that minimizes EB_n, and FB_n^* is used to approximate F_n^*. Then we have

Corollary 1: If there are r residual relays that have not received a message copy, where $\mu_{k_1}>\mu_{k_2}>...>\mu_{k_r}$, then FB_n^* also satisfies FB_n^*=$\{R_{k_1}, R_{k_2}, ..., R_{k_j}\}$, where $1 \leq j \leq r$.

This can be proved through the similar expressions of E_1 and EB_n. Note that EB_n can be written as follows:

$$EB_n=\frac{1}{s'_n}[\lambda_{max}+C\sum_{i\in F_n}\frac{\lambda_i^2}{\mu_i}] \quad \text{and} \quad C=\frac{(2\mu_{max}E_1^*)^{\frac{1}{2^{n-1}}}}{2} \tag{9}$$

In Eq. 9, parameter C is defined for presentation simplicity. Note that E_1 can be calculated as follows:

$$E_1 = \frac{1}{s_1}[1 + \sum_{i\in F_1}\frac{\lambda_i}{\mu_i}] \tag{10}$$

Since EB_n and E_n have similar expressions, Corollary 1 can be proved by a method that is similar to Theorem 1 (the proof is omitted due to the space limitation). In the next subsection, we will show and then analyze the whole algorithm.

C. Algorithm Design and Analysis

Again, the insight behind the forwarding set selection is quite simple and intuitive. The relay node R_k is added into the forwarding set, if passing a copy to R_k takes less delivery time than waiting for other feasible relays. Accordingly, we can iteratively select the relay, which has the smallest average delay to the destination among all residual relays, in the forwarding set. Since Eq. 5 is too complex to calculate, the upper bound of E_n in Eq. 9 is used to approximate the termination condition.

Considering that F_n *should include at least* n *relays (one copy for each relay)*, we add this constraint into the proposed forwarding set algorithm, as to avoid the possible premature termination brought by the approximated termination condition in Eq. 9. Then, the *multi-copy two-hop forwarding set selection* (MTFSS) algorithm is presented in Algorithm 1. The sorting of the residual relays takes a time complexity of $O(r \log r)$, and the greedy selection takes a time complexity of $O(r)$. So the total time complexity of MTFSS is $O(r \log r)$. Note that, updating E_1 or EB_n only takes constant time.

The whole routing algorithm is shown in terms of the source (the MTRA-Source in Algorithm 2) and the relays (the MTRA-Relay in Algorithm 3), respectively. The time complexity of the MTRA-Source is $O(nm \log m)$, since it calls for MTFSS n times. Moreover, this time complexity can be further reduced to $O(m \log m+nm)$, through sharing the sorting information among different rounds of calling MTFSS. In the next subsection, we will show an application of the MTRA, i.e., a multi-hop feature-based routing scheme.

III. Application

In this section, we introduce a multi-hop feature-based routing to relax the constraint that each copy in the two-hop routing can be forwarded at most twice. The basic idea is to use the social feature information to estimate the contact information. Then, the multi-hop feature-based is decomposed to a sequence of two-hop routings.

A. Basic Idea

In this subsection, we describe the basic idea of the feature-based routing scheme with the social feature information, where the contact frequencies (i.e., λ and μ in Fig. 2) are estimated by *feature distances* (i.e., the social feature dissimilarities between pairs of nodes [7]). This estimation is reasonable, since people with more common features contact each other more frequently [6]. The proverb that *birds of a feather flock together* also validates this fact. Let us consider an example in Fig. 3 with a three-dimensional feature space of gender, nationality, and position. The source is a female European student (on the left), while the destination is a male American professor (on the right). Then, the average delay of each link can be implicitly derived from their feature distances (i.e., the same gender/nationality/position or not). Note that the source has *direct* links to relays, while the relays have *indirect* links to the destination. Once the derivation is done, the routing in Fig. 3 is similar to the routing in Fig. 2. Therefore, we can apply the two-hop routing algorithm to select the forwarding set for this scenario, without using contact information. In the next subsection, we will show the detailed contact frequency (i.e., λ and μ) estimation process.

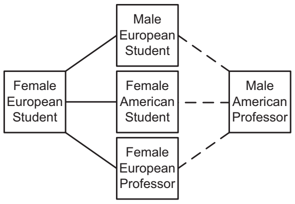


Fig. 3. Feature-based routing.

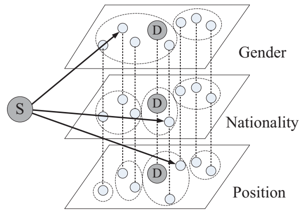


Fig. 4. Feature space routing.

B. Contact Frequency Estimation

Let c_l denote the cardinality size (the number of categories) of the l^{th} feature, e.g., the cardinality size of gender is 2 (i.e., male and female). As previously mentioned, contacts between the same gender should be more frequent than contacts between different genders. In other words, for a male node, the number of contacts with female nodes should be fewer than $\frac{1}{2}$ fraction of his total contacts. Now, let us go back to the more general case of the l^{th} feature. For a specified node, its number of contacts with a node that has a different category of the l^{th} feature is at most $\frac{1}{c_l}$. Here, we use the *upper bound estimation*: the contact frequency between people with different categories of the l^{th} feature is estimated to be $\frac{1}{c_l}$; the contact frequency between people with the same category of the l^{th} feature is estimated to be 1. Note that the estimated contact frequency is in a *relative* sense, rather than an absolute value.

Since the feature space contains more than one feature, we need to estimate the contact frequency between two people with δ different features, where $l \in [1, \delta]$. Therefore, we use

$$\delta! \times \prod_{l=1}^{\delta} \frac{1}{c_l} \tag{11}$$

to estimate their contact frequency (i.e., parameters λ and μ in Fig. 2). In Eq. 11, the former part $\delta!$ means the whole sequence of the δ features, while the latter part $\prod_{l=1}^{\delta} \frac{1}{c_l}$ represents the contact frequency of each sequence of the δ features. Let us go back to the example in Fig. 3. Assume there are δ=3 features in total: Gender=$\{$male, female$\}$, Nationality=$\{$African, American, European$\}$ and Position=$\{$student, professor, labour, manager$\}$. Then, the direct contact frequency (i.e., λ_0) between source and destination is estimated as $[3! \times (\frac{1}{2} * \frac{1}{3} * \frac{1}{4})] = 0.25$, since all three features are different.

In addition, a problem is that people may not provide all of their social feature information, due to privacy issues. Our algorithm supports the condition of incomplete information, e.g., the gender information is unknown. At this time, it is regarded as an unknown gender category, which is different from the gender categories of all the other people. Meanwhile, the unknown category does not contribute to the cardinality size. In the next subsection, we will describe the whole feature-based routing scheme, with its performance analysis.

C. Feature Space Routing

Here, we denote the feature-based routing scheme as the *feature space routing*. The feature space corresponding to Fig. 3 is shown in Fig. 4, where each node appears three times in three layers (each layer represents a social feature). Nodes with similar features can be viewed as in the same "community" [6] of a feature, resulting in frequent contacts. The source has direct links to relays, while the relays have indirect links to the destination. The contact frequencies are estimated through the feature distances, as described in the previous subsection. Now, the source can distribute copies, according to the two-hop routing algorithm in Section II.

Since links from relays to destination are indirect, the data delivery is no longer in two hops. Once receiving a message copy from source, each message holder uses the single-copy two-hop routing algorithm to forward the message copy, until the message reaches the destination, or a node that has totally the same features as the destination. If the latter condition happens, then this node will hold the copy until meeting with the destination (nodes with the same features meet frequently). Note that, only the source uses MTRA to distribute copies to relays, while all the relays *iteratively* use single-copy two-hop routing algorithm (i.e., MTRA with one copy) to forward copies. Therefore, a multi-hop routing can be approximated through a sequence of two-hop routings with a direct first hop link and an indirect second hop link. Each indirect link can be iteratively decomposed by a sequence of two-hop routings.

Moreover, the feature space algorithm also *limits the number of forwarding times*, where each copy is forwarded at most δ+1 times. This is because the actual relays should have at least one more common feature with the destination than the message holder (except for the last relay that may have all the same features with the destination). Otherwise, this relay is not selected in the forwarding set of the message holder. Therefore, the resource consumption of the feature space routing is also bounded, although the routing takes more than two hops.

IV. Simulation

In this section, extensive real trace–driven simulations are conducted to evaluate the performance of the proposed algorithms. The evaluation results are shown from different perspectives to provide insightful conclusions.

A. Real Trace–Driven

Our simulations are driven by both synthetic and real traces. A synthetic trace and the Intel trace [11] are used to test the performance of MTRA, which requires that the destination is within the two-hop neighbor of the source. In the synthetic trace, the source and the destination are fixed with a total of 30 available relays between them, while no direct connection exists between the source and destination. λ and μ are assigned to be uniformly distributed numbers in $[0, 10^{-1}]$ and $[0, 10^{-2}]$, respectively. The unit of λ and μ is min^{-1}. The reason why we assign λ to be 10 times bigger than μ is: (1) in testbeds where $\mu \gg \lambda$, the optimal forwarding set would include almost all relays; (2) in testbeds where $\mu \ll \lambda$, only the best n relays should be selected; (3) in the assigned testbed, the optimal forwarding set would include appropriate numbers of relays, which serves the purpose of the test. As for the Intel trace, it is collected by assigning people to carry mobile devices (iMotes) for several days. The iMotes are called internal nodes, while other Bluetooth devices are called external devices. Only internal nodes are employed in our simulation, the number of which is 9. In this trace, each pair of nodes can be connected in two hops. Here, we do not use large-scale real traces, since we want to guarantee the constraint that the destination is a two-hop neighbor of the source (no delivery rate issue).

The Optimal Forwarding Set By Exhaustion and The Forwarding Set By MTFSS		
$n = 3$	$n = 2$	$n = 1$
$[R_1 R_2 R_3]$	R_1 gets 1^{st} copy $\Rightarrow [R_2 R_3]$	R_2 gets 2^{nd} copy $\Rightarrow [R_3]$
		R_3 gets 2^{nd} copy $\Rightarrow [R_2]$
	R_2 gets 1^{st} copy $\Rightarrow [R_1 R_3]$	R_1 gets 2^{nd} copy $\Rightarrow [R_3]$
		R_3 gets 2^{nd} copy $\Rightarrow [R_1]$
	R_3 gets 1^{st} copy $\Rightarrow [R_1 R_2]$	R_1 gets 2^{nd} copy $\Rightarrow [R_2]$
		R_2 gets 2^{nd} copy $\Rightarrow [R_1]$

The Forwarding Set By Repeated STFSS		
$n = 3$	$n = 2$	$n = 1$
$[R_1]$	R_1 gets 1^{st} copy $R_1 \Rightarrow [R_2]$	R_2 gets 2^{nd} copy $\Rightarrow [R_3]$

Fig. 5. A case study for testing the bound tightness.

The extended feature space routing scheme is tested in the MIT trace [12] and the Infocom 2006 trace [13]. The MIT trace records the contacts between the participants on the campus, where we extract 5 social features from it: neighborhood, daily commute, hangouts, affiliation, and research group. The MIT trace contains 106 nodes, while 12 nodes without social feature information are removed. Then, the Infocom 2006 trace is a conference contact trace, and we extract 6 social features from this dataset: affiliation, city, nationality, language, country, and position. Data is collected through iMotes, and again, only 98 internal nodes are used. Nodes without contact history or feature information are removed, with 61 nodes remaining. For these two traces, if a node has multiple properties for a feature, only the primary property is selected. For example, if a person can speak both French and German (the language feature), then only his mother tongue is selected.

For all real traces, we randomly generate pairs of source and destination for the performance tests. Simulation results are averaged over 1,000,000 times. Meanwhile, 1000 minutes are used as the data delivery deadline for all tests. If the destination is not achieved before the deadline, then the data delivery is viewed as having failed, while the deadline serves as the delivery delay. In addition, a metric called $Gain\,Ratio$ [14] is used to further analyze our algorithms (i.e, the ratio of delay reduction brought by one more message copy):

$$Gain\,Ratio(n) = \frac{Delay_n - Delay_{n+1}}{Delay_n} \tag{12}$$

B. Algorithms for Comparison

For MTRA, we assign the following algorithms for comparison. (1) Infinite Copies, where the source has infinite copies with F_n={All Relays}. The source forwards a copy to the relay node if inter-meeting is available. Infinite Copies shows the minimum data delivery delay of two-hop routing algorithms. (2) All New Paths, where the source also forwards one message copy to any inter-meeting relay nodes (F_n={All Relays}). However, the number of copies is limited to be n rather than infinite. (3) Repeated STRA, the source routes the n copies using single-copy two-hop routing recursively, i.e., F_n=F_1^*. In Repeated STRA, F_1^* is calculated based on current residual relays for each round. Here, we focus on comparing the difference between the forwarding-set-based two-hop routing algorithms, as to observe the performance of MTFSS. Therefore, routing algorithms, such as spray and wait, delegation forwarding [15], are not included. Note that the performance bound of MTFSS has been analyzed in Eqs. 7.

As for feature space routing algorithm (FSR-MTRA for short), we use four algorithms for comparison. (1) Epidemic, where the nodes continuously replicate and transmit messages to newly discovered contacts that do not already possess a copy. Epidemic represents the minimum data delivery delay of all routing algorithms. (2) (Binary) Spray and Wait, where the data delivery is composed of a spray phase and wait phase; during the spray phase, the source of the message is responsible for delivering copies to relays. When a relay receives the copy, it enters the wait phase, where the relay simply holds the copy until the destination is encountered directly. (3) SimBet [8], where the relays are selected according to similarity and betweenness. Here, we only use similarity information due to local information. Each message holder will give a copy to a inter-meeting relay if this relay does not hold a copy and has shorter feature distance with the destination. Note that, only source holds multiple copies. (4) The feature space routing that is based on Repeated STRA (FSR-RSTRA for short). The only difference is that we use Repeated STRA to determine the forwarding set, rather than using MTRA. The former two algorithms do not need any network information, while the latter two algorithms only need feature information. In addition, algorithms such as BubbleRap are not included since additional community information is required.

C. Bound Tightness

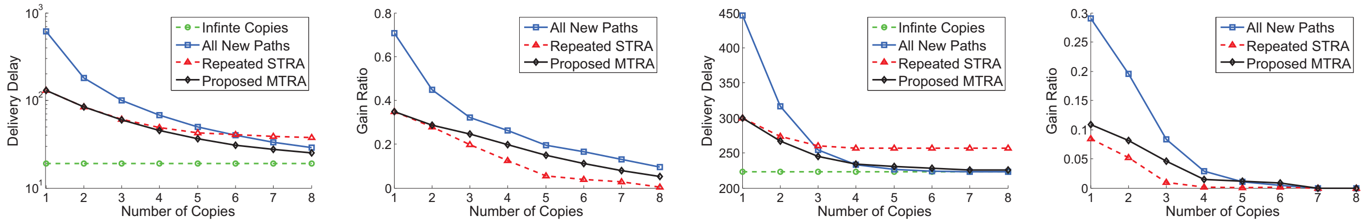
To test the bound tightness of MTFSS, we show a case study in this subsection. A small test set is synthetically generated, where we have m=5 source-relay-destination paths and n=3 message copies. As aforementioned, λ and μ are assigned to be uniformly distributed numbers in $[0, 10^{-1}]$ and $[0, 10^{-2}]$ as follows (corresponding to relays $[R_1, ..., R_5]$):

$$\lambda = [6.92, 1.13, 5.27, 9.06, 8.15] \times 10^{-2}$$
$$\mu = [0.93, 0.74, 0.30, 0.28, 0.09] \times 10^{-2}$$

The test results have been shown in Fig. 5, where the parameter n denotes the number of copies hold by the source. MTFSS finds out the optimal forwarding set in this case study, while a huge gap exists between the repeated STRA and the optimal one. Repeated STRA fails to reserve relays for the remaining copies, leading to a great performance degradation.

D. Evaluation Results

The evaluation results of two-hop routing algorithms are shown in Fig. 6, where we focus on the performance gap between MTRA and other forwarding-set-based algorithms. The delivery rate is not considered, since the source and destination are always connected in two hops (as previously mentioned). In the view of data delivery delay, MTRA outperforms all algorithms, except for the Infinite Copies that represents the limitation of the two-hop routing. The performance gap between MTRA and Repeated STRA becomes larger with respect to the number of message copies. In the view of $Gain\,Ratio$, all the algorithms satisfy *diminishing return*. As previously analyzed, for MTRA, the delay reduction contributed by an addition copy exponentially decays with respect to the number of copies. The routing algorithm of All New Paths has a high gain ratio at first, and then decays quickly with the number of copies n. This is because the copies sent by it may end up choosing unqualified relays.



(a) Delivery Delay (synthetic trace) (b) Gain Ratio (synthetic trace) (c) Delivery Delay (Intel trace) (d) Gain Ratio (Intel trace)

Fig. 6. Multi-copy two-hop routing. In this setting, we focus on the performance gap between different forwarding set selection methods.

The evaluation results of the feature space routing scheme, which is based on the two-hop routing schemes, are shown in Fig. 7. Here, Epidemic and Spray & Wait are not related to the feature information, while FSR-RSTRA, FSR-MTRA, and SimBet are feature-based routing algorithms. For both delivery delay and delivery ratio, FSR-MTRA outperforms FSR-RSTRA and SimBet, meaning FSR-MTRA has a better and more reasonable forwarding set selection. Although FSR-MTRA additionally utilizes the feature information, it has a less resource consumption. This is because each forwarding happens between nodes with different features, while Spray & Wait always distributes copies among nodes with the same features. In the view of *Gain Ratio*, all algorithms also satisfy diminishing return, as previously analyzed.

V. Conclusion

In this paper, we introduce the concept of forwarding set for opportunistic two-hop routings. A multi-copy two-hop routing algorithm (MTRA) is proposed with a performance bound. All the forwarding sets for the n copies can be efficiently determined with a time complexity of $O(m \log m + nm)$, where m is the number of available relays. Then, MTRA is applied to a feature space routing scheme, where the contact frequencies are estimated by feature distances. Simulation results show competitive performances of the proposed algorithms, which fully utilize the opportunistic nature of MSNs.

References

[1] Z. Zhang, "Routing in intermittently connected mobile ad hoc networks and delay tolerant networks: overview and challenges," *IEEE Communications Surveys & Tutorials*, vol. 8, no. 1, pp. 24–37, Mar. 2006.

[2] V. Conan, J. Leguay, and T. Friedman, "Fixed point opportunistic routing in delay tolerant networks," *IEEE J. Sel. A. Commun.*, vol. 26, no. 5, pp. 773–782, Jun. 2008.

[3] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, pp. 477–486, Aug. 2002.

[4] W. Zhao, M. Ammar, and E. Zegura, "Controlling the mobility of multiple data transport ferries in a delay-tolerant network," in *Proc. of IEEE INFOCOM 2005*, pp. 1407–1418.

[5] H. Zhu, S. Chang, M. Li, K. Naik, and S. Shen, "Exploiting temporal dependency for opportunistic forwarding in urban vehicular networks," in *Proc. of IEEE INFOCOM 2011*, pp. 2192–2200.

[6] J. Wu and Y. Wang, "Social feature-based multi-path routing in delay tolerant networks," in *Proc. of IEEE INFOCOM 2012*, pp. 1368–1376.

[7] K. Wei, X. Liang, and K. Xu, "A survey of social-aware routing protocols in delay tolerant networks: Applications, taxonomy and design-related issues," *IEEE Communications Surveys Tutorials*, vol. PP, no. 99, pp. 1–23, May 2013.

[8] E. M. Daly and M. Haahr, "Social network analysis for routing in disconnected delay-tolerant manets," in *Proc. of ACM MobiHoc 2007*, pp. 32–40.

[9] X. Tie, A. Venkataramani, and A. Balasubramanian, "R3: robust replication routing in wireless networks with diverse connectivity characteristics," in *Proc. of ACM MobiCom 2011*, pp. 181–192.

[10] A. Balasubramanian, B. N. Levine, and A. Venkataramani, "Replication routing in dtns: a resource allocation approach," *IEEE/ACM Trans. Netw.*, vol. 18, no. 2, pp. 596–609, Apr. 2010.

[11] J. Scott, R. Gass, J. Crowcroft, P. Hui, C. Diot, and A. Chaintreau, "CRAWDAD trace cambridge/haggle/imote/intel (v. 2006-01-31)," Downloaded from http://crawdad.cs.dartmouth.edu/cambridge/haggle/imote/intel, Jan. 2006.

[12] N. Eagle, A. S. Pentland, and D. Lazer, "Inferring friendship network structure by using mobile phone data," *PNAS*, vol. 106, no. 36, pp. 15 274–15 278, Aug. 2009.

[13] J. Scott, R. Gass, J. Crowcroft, P. Hui, C. Diot, and A. Chaintreau, "CRAWDAD trace cambridge/haggle/imote/ infocom2006 (v. 2009-05-29)," Downloaded from http://crawdad.cs.dartmouth.edu/cambridge/haggle/imote/infocom2006, 2009.

[14] Y. Zhao and J. Wu, "On the construction of the minimum cost content-based publish/subscribe overlays," in *Proc. of IEEE SECON 2011*, pp. 476–484.

[15] V. Erramilli, M. Crovella, A. Chaintreau, and C. Diot, "Delegation forwarding," in *Proc. of ACM MobiHoc 2008*, pp. 251–260.

Appendix

A. PDF Expression Simplification

First, let us solve $g^i_{n-1}(t)$ from Eq. 2:

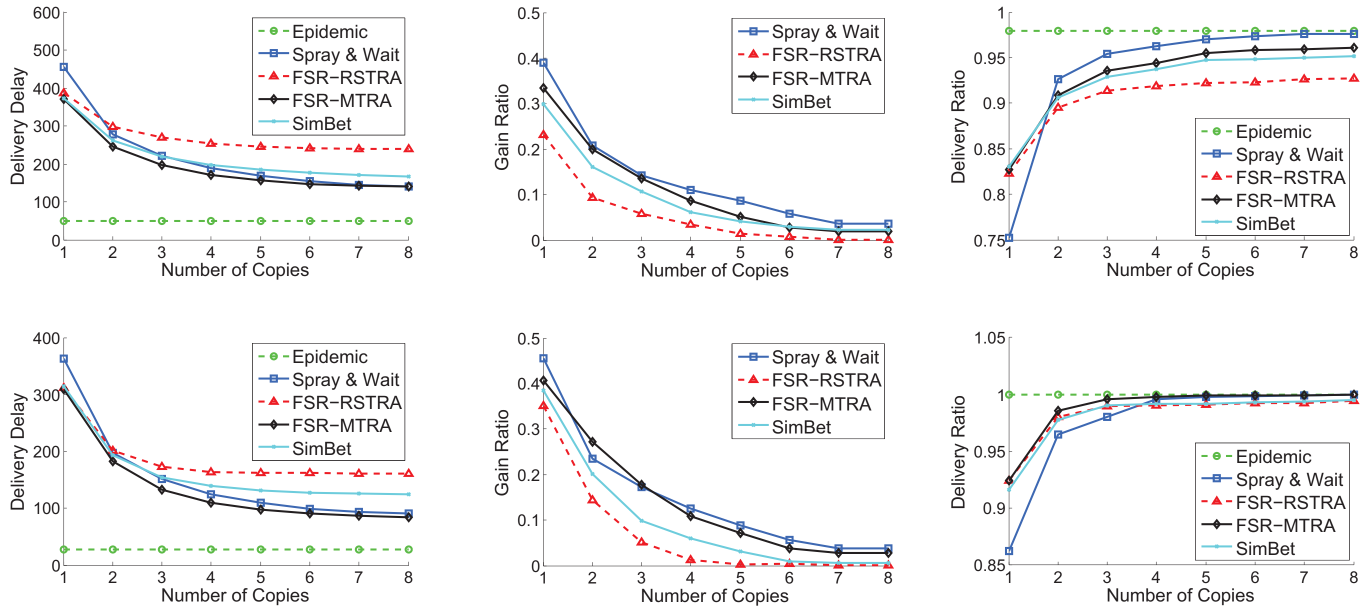
$$\begin{aligned} g^i_{n-1}(t) &= [G^i_{n-1}(t)]' = \{1 - e^{-\mu_i t}[1 - H_{n-1}(t)]\}' \\ &= \mu_i e^{-\mu_i t}[1 - H_{n-1}(t)] + e^{-\mu_i t} h_{n-1}(t) \end{aligned} \tag{13}$$

Then, Eq. 1 is changed to

$$\begin{aligned} h_n(t) &= \int_0^t \sum_{i\in F_n} \{[\lambda_i e^{-s_n T}][g^i_{n-1}(t-T)]\} dT \\ &= \int_0^t \sum_{i\in F_n} \{[\lambda_i e^{-s_n (t-T)}][g^i_{n-1}(T)]\} dT \\ &= e^{-s_n t} \int_0^t \sum_{i\in F_n} [\lambda_i e^{s_n T} g^i_{n-1}(T)] dT \end{aligned} \tag{14}$$

Substitute $g^i_{n-1}(T)$ in Eq. 13 with Eq. 14,

$$\begin{aligned} h_n(t) = {} & e^{-s_n t} \int_0^t [\sum_{i\in F_n} \lambda_i \mu_i e^{(s_n - \mu_i)T}] dT \\ & - e^{-s_n t} \int_0^t [\sum_{i\in F_n} \lambda_i \mu_i e^{(s_n - \mu_i)T} H_{n-1}(T)] dT \\ & + e^{-s_n t} \int_0^t [\sum_{i\in F_n} \lambda_i e^{(s_n - \mu_i)T} h_{n-1}(T)] dT \end{aligned} \tag{15}$$



(a) Delivery Delay (b) Gain Ratio (c) Delivery Ratio

Fig. 7. The feature space routing in MIT trace (top) and Infocom 2006 trace (bottom). Here, Epidemic and Spray & Wait are not related to the feature information, while FSR-RSTRA, FSR-MTRA, and SimBet are feature-based routing algorithms.

Thus, we have

$$\begin{aligned}\frac{h_n(t)}{e^{-s_n t}} =& \sum_{i\in F_n} \lambda_i\mu_i \int_0^t e^{(s_n-\mu_i)T}dT \\ &- \sum_{i\in F_n} \lambda_i\mu_i \int_0^t e^{(s_n-\mu_i)T} H_{n-1}(T)dT \\ &+ \sum_{i\in F_n} \lambda_i \int_0^t e^{(s_n-\mu_i)T} h_{n-1}(T)dT \qquad (16)\end{aligned}$$

The sub-factor of the second item in Eq. 16 is

$$\begin{aligned}&\int_0^t e^{(s_n-\mu_i)T} H_{n-1}(T)dT \\ =&\int_0^t e^{(s_n-\mu_i)T}[\int_0^T h_{n-1}(\tau)d\tau]dT \\ =&\int_0^t h_{n-1}(\tau)[\int_\tau^t e^{(s_n-\mu_i)T}dT]d\tau \\ =&\frac{1}{s_n-\mu_i}\int_0^t h_{n-1}(\tau)[e^{(s_n-\mu_i)t} - e^{(s_n-\mu_i)\tau}]d\tau \qquad (17)\end{aligned}$$

Back to Eq. 16 with the substitution in Eq. 17, we have

$$\begin{aligned}\frac{h_n(t)}{e^{-s_n t}} =& \sum_{i\in F_n} \frac{\lambda_i\mu_i}{s_n-\mu_i}[e^{(s_n-\mu_i)t} - 1] \\ &- \sum_{i\in F_n} \frac{\lambda_i\mu_i}{s_n-\mu_i}\int_0^t h_{n-1}(\tau)e^{(s_n-\mu_i)t}d\tau \\ &+ \sum_{i\in F_n}[\lambda_i + \frac{\lambda_i\mu_i}{s_n-\mu_i}]\int_0^t e^{(s_n-\mu_i)T}h_{n-1}(T)dT \qquad (18)\end{aligned}$$

Simplifying Eq. 18, we have:

$$\begin{aligned}h_n(t) =& \sum_{i\in F_n}\frac{\lambda_i\mu_i}{s_n-\mu_i}[e^{-\mu_i t} - e^{-s_n t} - e^{-\mu_i t}H_{n-1}(t)] \\ &+ \sum_{i\in F_n}\frac{\lambda_i s_n e^{-s_n t}}{s_n-\mu_i}\int_0^t e^{(s_n-\mu_i)T}h_{n-1}(T)dT \qquad (19)\end{aligned}$$

B. Expected Delay

Through $h_n(t)$ in Eq. 4, E_n can be calculated by

$$\begin{aligned}E_n =& \int_0^\infty t h_n(t)dt \\ =& \sum_{i\in F_n}\frac{\lambda_i\mu_i}{s_n-\mu_i}\int_0^\infty t[e^{-\mu_i t} - e^{-s_n t}]dt \\ &- \sum_{i\in F_n}\frac{\lambda_i\mu_i}{s_n-\mu_i}\int_0^\infty t e^{-\mu_i t}H_{n-1}(t)dt \\ &+ \sum_{i\in F_n}\frac{\lambda_i s_n \int_0^\infty t e^{-s_n t}\int_0^t e^{(s_n-\mu_i)T}h_{n-1}(T)dtdT}{s_n-\mu_i} \qquad (20)\end{aligned}$$

For the three terms in Eq. 20, the first one equals $\frac{1}{s_n}[1 + \sum_{i\in F_n}\frac{\lambda_i}{\mu_i}]$. The second one in Eq. 20 is:

$$\begin{aligned}&\sum_{i\in F_n}\frac{\lambda_i\mu_i}{s_n-\mu_i}\int_0^\infty t e^{-\mu_i t}H_{n-1}(t)dt \\ =&\sum_{i\in F_n}\frac{\lambda_i\mu_i}{s_n-\mu_i}\int_0^\infty t e^{-\mu_i t}[\int_0^t h_{n-1}(\tau)d\tau]dt \\ =&\sum_{i\in F_n}\frac{\lambda_i\mu_i}{s_n-\mu_i}\int_0^\infty h_{n-1}(\tau)[\int_\tau^\infty t e^{-\mu_i t}dt]d\tau \\ =&\sum_{i\in F_n}\frac{\lambda_i}{s_n-\mu_i}\int_0^\infty h_{n-1}(\tau)[\tau e^{-\mu_i\tau} + \frac{e^{-\mu_i\tau}}{\mu_i}]d\tau \qquad (21)\end{aligned}$$

The third term in Eq. 20 is

$$\sum_{i\in F_n}\frac{\lambda_i s_n}{s_n-\mu_i}\int_0^\infty te^{-s_n t}[\int_0^t e^{(s_n-\mu_i)T}h_{n-1}(T)dT]dt$$
$$=\sum_{i\in F_n}\frac{\lambda_i s_n}{s_n-\mu_i}\int_0^\infty e^{(s_n-\mu_i)T}h_{n-1}(T)[\int_T^\infty te^{-s_n t}dt]dT$$
$$=\sum_{i\in F_n}\frac{\lambda_i}{s_n-\mu_i}\int_0^\infty e^{(s_n-\mu_i)T}h_{n-1}(T)[Te^{-s_n T}+\frac{e^{-s_n T}}{s_n}]dT$$
$$=\sum_{i\in F_n}\frac{\lambda_i}{s_n-\mu_i}\int_0^\infty h_{n-1}(T)[Te^{-\mu_i T}+\frac{e^{-\mu_i T}}{s_n}]dT \quad (22)$$

Combining the three terms in Eq. 20, we have

$$E_n=\frac{1}{s_n}[1+\sum_{i\in F_n}\frac{\lambda_i}{\mu_i}]$$
$$+\sum_{i\in F_n}\frac{\lambda_i}{s_n-\mu_i}\int_0^\infty h_{n-1}(t)[\frac{e^{-\mu_i T}}{s_n}-\frac{e^{-\mu_i T}}{\mu_i}]dT$$
$$=\frac{1}{s_n}\{1+\sum_{i\in F_n}\frac{\lambda_i}{\mu_i}[1-\int_0^\infty e^{-\mu_i T}h_{n-1}(T)dT]\} \quad (23)$$

Integration by parts in Eq. 23, we have

$$E_n=\frac{1}{s_n}[1+\sum_{i\in F_n}\frac{\lambda_i}{\mu_i}]-\sum_{i\in F_n}\frac{\lambda_i}{s_n}\int_0^\infty e^{-\mu_i t}H_{n-1}(t)dt \quad (24)$$

C. Proof of Theorem 1

Obviously, $F_n \nsubseteq \emptyset$, otherwise the data delivery delay is infinite. Note that the paths satisfy $\mu_{k_1}>\mu_{k_2}>...>\mu_{k_r}$. Let $\overline{F_n^*}=F_n^*\cup R_k$ denote the collection of the optimal forwarding set, F_n^*, and an addition relay R_k that is not in F_n^*. And let $\overline{E_n^*}$ denote the corresponding expected delivery delay of $\overline{F_n^*}$. According to Eq. 5, then we have (note that $s_n=\sum_{i\in F_n}\lambda_i$)

$$\overline{E_n^*}-E_n^*=\frac{\lambda_k}{s_n+\lambda_k}\{[\frac{1}{\mu_k}-\int_0^\infty e^{-\mu_k t}H_{n-1}(t)dt]-E_n^*\}$$
$$=\frac{\lambda_k}{s_n+\lambda_k}\{\int_0^\infty e^{-\mu_k t}[1-H_{n-1}(t)]dt-E_n^*\} \quad (25)$$

Here, $[1-H_{n-1}(t)]$ monotonically decreases with t, since $H_{n-1}(t)$ is a CDF that monotonically increases with t. So the function $Y(\mu_k)=\int_0^\infty e^{-\mu_k t}[1-H_{n-1}(t)]dt$ monotonically decreases with μ_k. Note that, essentially, $H_{n-1}(t)$ varies for different μ_k, since the residual relays are uncertain (the actual relay of the current copy is uncertain). However, we ignore this difference when $r\gg 1$, i.e., the function $Y()$ is regarded as a fixed function for different μ_k. Due to the optimality assumption, we have $\overline{E_n^*}-E_n^*>0$, meaning $Y(\mu_k)>E_n^*$. Let $Y^{-1}()$ denote the inverse function of $Y()$. Then, we have $\mu_k<Y^{-1}(E_n^*)$ according to the monotone of $Y()$. Obviously, $Y^{-1}(E_n^*)$ is a certain constant for an existing routing model. Therefore, all paths with $\mu<Y^{-1}(E_n^*)$ are not selected, i.e., all paths with $\mu\geq Y^{-1}(E_n^*)$ are selected. Accordingly, we have $F_n^*=\{R_{k_1},R_{k_2},...,R_{k_j}\}$ for a specified j in $[1,r]$. Meanwhile, when n=1, we have exactly $F_1^*=\{R_{k_1},R_{k_2},...,R_{k_j}\}$, since $H_{n-1}(t)=0$ is exactly the same for different relays.

D. Proof of Theorem 2

Let us begin with a lemma from Section 3.2 in [9]:

Lemma 1: Assume $h(t)/H(t)$ is the PDF/CDF of a random variable T, then the mathematical expectation of T is

$$E=\int_0^\infty th(t)dt=\int_0^\infty[1-H(t)]dt \quad (26)$$

This lemma can be proved, first, by rewriting $t=\int_0^t d\tau$, and second, by exchanging the integral order of t and τ. Let us go back to the proof of Theorem 2:

$$E_n=\frac{1}{s_n}[1+\sum_{i\in F_n}\frac{\lambda_i}{\mu_i}]-\sum_{i\in F_n}\frac{\lambda_i}{s_n}\int_0^\infty e^{-\mu_i t}H_{n-1}(t)dt$$
$$=\frac{1}{s_n}+\sum_{i\in F_n}\frac{\lambda_i}{s_n}\int_0^\infty e^{-\mu_i t}[1-H_{n-1}(t)]dt \quad (27)$$

According to the Cauchy-Schwarz inequality (integral formula), and Lemma 1, we have

$$E_n\leq\frac{1}{s_n}+\sum_{i\in F_n}\frac{\lambda_i}{s_n}\sqrt{\int_0^\infty[e^{-\mu_i t}]^2dt\int_0^\infty[1-H_{n-1}(t)]^2dt}$$
$$\leq\frac{1}{s_n}+\sum_{i\in F_n}\frac{\lambda_i}{s_n}\sqrt{\int_0^\infty e^{-2\mu_i t}dt\int_0^\infty[1-H_{n-1}(t)]dt}$$
$$=\frac{1}{s_n}[1+\sum_{i\in F_n}\frac{\lambda_i\sqrt{E_{n-1}}}{\sqrt{2\mu_i}}] \quad (28)$$

E. Upper Bound Simplification

Since $E_n\gg 1/s_n$, according to Eq. 7, we have

$$E_n-\frac{1}{s_n}\leq\frac{1}{s_n}\sum_{i\in F_n}\frac{\lambda_i}{\sqrt{2\mu_i}}\sqrt{E_{n-1}}$$
$$\approx\frac{1}{s_n}\sum_{i\in F_n}\frac{\lambda_i}{\sqrt{2\mu_i}}[E_{n-1}-\frac{1}{s_{n-1}}]^{\frac{1}{2}} \quad (29)$$

For $m\gg n$ and $n\geq 2$, we have

$$\frac{1}{s_n}[\frac{\sum_{i\in F_n}\lambda_i}{\sqrt{2\mu_i}}]\approx\frac{1}{s_1}[\frac{\sum_{i\in F_1}\lambda_i}{\sqrt{2\mu_i}}] \quad (30)$$

Then, Eq. 29 can be approximated to be

$$E_n-\frac{1}{s_n}\leq[\frac{1}{s_n}\sum_{i\in F_n}\frac{\lambda_i}{\sqrt{2\mu_i}}]^{1+\frac{1}{2}+...+\frac{1}{2^{n-2}}}[E_1-\frac{1}{s_1}]^{\frac{1}{2^{n-1}}}$$
$$\approx[\frac{1}{s_n}\sum_{i\in F_n}\frac{\lambda_i}{\sqrt{2\mu_i}}]^{2-\frac{1}{2^{n-2}}}[E_1^{\frac{1}{2^{n-1}}}] \quad (31)$$

Since $\lambda\ll 1$, $\mu\ll 1$, and $1/\sqrt{2\mu_i}>1$, we have

$$\frac{[\sum_{i\in F_n}\frac{\lambda_i}{\sqrt{2\mu_i}}]^2}{[\sum_{i\in F_n}\lambda_i]^2}\approx\frac{\sum_{i\in F_n}\frac{\lambda_i^2}{2\mu_i}}{\sum_{i\in F_n}\lambda_i^2}=\frac{\sum_{i\in F_n}\frac{\lambda_i^2}{2\mu_i}}{s'_n} \quad (32)$$

Finally,

$$E_n\leq\frac{1}{s_n}+[\frac{1}{s'_n}\sum_{i\in F_n}\frac{\lambda_i^2}{2\mu_i}]^{1-\frac{1}{2^{n-1}}}[E_1^{\frac{1}{2^{n-1}}}]$$
$$=\frac{s'_n/s_n}{s'_n}+[\frac{1}{s'_n}\sum_{i\in F_n}\frac{\lambda_i^2}{2\mu_i}][\frac{E_1}{\frac{1}{s'_1}\sum_{i\in F_n}\frac{\lambda_i^2}{2\mu_i}}]^{\frac{1}{2^{n-1}}}$$
$$<\frac{\lambda_{max}}{s'_n}+[\frac{1}{s'_n}\sum_{i\in F_n}\frac{\lambda_i^2}{2\mu_i}][2\mu_{max}E_1]^{\frac{1}{2^{n-1}}} \quad (33)$$

where λ_{max} and μ_{max} is the max value of the λ and μ of all initial paths (excluding $\mu_0=\infty$).