Supplemental Material of "Efficient Virtual Backbone Construction without a Common Control Channel in Cognitive Radio Networks"

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I. Benefits nd Challenges

In this section, we intuitively discuss the characteristics of CRNs and the special benefits brought by virtual backbones, compared to other networks. Then, we present the challenges of constructing virtual backbones in CRNs.

A. Benefits of Virtual Backbones in CRNs

The interference of a single link in traditional wireless networks usually comes from other links. The interfered sender can compete with other nodes fairly, using existing MAC protocols. However, in CRNs, interference to a single link consists of two parts: 1) interference from other SUs; 2) interference from PUs. The interfered nodes in CRNs can compete equally with other nodes for channel access, as is true in other networks. However, they cannot compete with PUs, since the PUs have privileged access on channels. Their data transmission cannot be interfered with by SUs. Therefore, for a single link in CRNs, if it is interfered by a suddenly active PU, it cannot compete with that PU. If the interfered link still wants to transmit on that channel, it has to wait for the PU's data transmission to be finished. However, the signal duration of PUs usually lasts longer, which means the interfered link is broken for a long time. Therefore, the dynamics of channel availabilities in CRNs are more special than other wireless networks. With virtual backbones, the area routing scheme is implementable and very effective in reducing the influences caused by unpredictable PU activities.

B. Challenges of Constructing Virtual Backbones in CRNs

Given the special benefits brought to CRNs by virtual backbones, the remaining problem is how to construct virtual backbones in CRNs? Or, is the method exactly the same as that used to construct virtual backbones in other wireless networks?

The answer is no. The key challenge faced in constructing virtual backbones in CRNs is still the dynamics in channel availabilities. The existing approaches of building virtual backbones rely on a common channel to exchange control messages, no matter if it is exchanging information among 1-hop or 2-hop neighbors, or neighbors with even further hop distance. However, in CRNs, the original objective of building virtual backbones is to solve the data transmission without stable channels. This means that, during the virtual backbone construction process, there is no stable common channel in which nodes can exchange information. Therefore, we need to come up with a virtual backbone construction scheme that fits the CRNs.

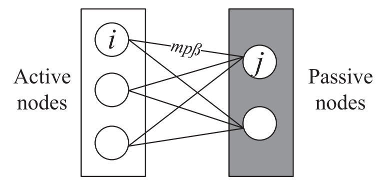

Fig. 1. The probability of connection as a bipartite graph.

II. Proof of Theorem 1

Proof: The probability that i can meet j on the kth ($1 \leq k \leq m$) channel among the m available channels is βp, i.e., if node j selects the kth channel as the home channel, and the kth channel is available to node i. The total probability that node i meets node j in any of the m channels is

$$\sum\nolimits_{k=1}^{m} \beta p = m\beta p. \tag{1}$$

The $\tilde{N}$ passive nodes and the $\bar{N}$ active nodes in the cell of nodes i and j form a complete bipartite graph (as shown in Fig. 1) with the active nodes on one side and passive nodes on the other side. Let the cost of a link denote the *link connection probability*, which is the probability that two end nodes meet each other in the self-organization. In Eq. (1), the connection probability of every link is $m\beta p$, since (1) does not depend on i or j. The active node i and passive node j are disconnected only if all links of a cut are disconnected. Let n denote the number of links of the cut, then the probability that nodes i and j are disconnected due to this cut is $(1 - m\beta p)^n$. We can see that this *nodes disconnection probability* increases when n decreases. Thus, the disconnection probability is maximized if the links of a mincut are disconnected. For a complete bipartite with $\tilde{N}$ passive nodes on the left and $\bar{N}$ active nodes on the right, the cardinality of the mincut between nodes i and j is $\min(d_i, d_j) = \min(\tilde{N}, \bar{N})$, where d_i is the degree of node

i. Therefore, the disconnection probability, denoted as P_d, is upper-bounded as

$$P_d \leq (1 - m\beta p)^{\min(\tilde{N}, \bar{N})}.$$

Accordingly, the connection probability between the active node i and passive node j through all possible routes in the bipartite graph is

$$P_s = 1 - P_d \geq 1 - (1 - m\beta p)^{\min(\tilde{N}, \bar{N})}.$$

■

III. Proof of Theorem 2

Proof: For a node i, after the cluster heads are selected in each cell, i must be covered. This is because, based on our cell division, i must belong to a cell c_k. Then in c_k, there exists a node h, $h \in H$, ($h = i$ or $h \neq i$), that is a cluster head in c_k and is connected to i. Thus, i is covered. ■

IV. Proof of Theorem 3

Proof: In [1], the authors proved that their approach ensures connectivity and coverage. What we need to prove here is that the backbone nodes B' selected in [1] are still connected using B. The difference is that their marking process and pruning rule is performed for all cluster heads within r_2, and ours is performed for all cluster heads in adjacent cells. For a cluster head h_1, it conducts the marking process and pruning rule for adjacent cells. Cluster heads outside the adjacent cells of h, but within the range of r_2, are connected because none of them are removed. For cluster heads within the adjacent cells of h, they are connected, since the results are the same as in [1]'s approach; this ensures the connectivity and the coverage. ■

V. Analysis of Capacity between an Active Node and a Passive Node

In this section, we analyze the capacity between an active node and a passive node in a single area. This is for the analysis of the capacity in a forwarding area, which is in the next section. The analysis results can be used to guide the routing algorithms under our model, for example, to find a route with the largest capacity, and also to decrease congestion.

Let node j be an active node and node i be a passive node. The two nodes are in the same area. Let $\tilde{N}$ denote the number of passive nodes in the area, and $\bar{N}$ denote the number of active nodes in the area. Let M denote the number of channels. First, we introduce two lemmas.

***Lemma** 1:* Let α denote the success probability for node j to estimate the home channel of node i. Then α is given as

$$\alpha = \frac{p}{2-p}\left(1 - (1-p)^{2M}\right), \tag{2}$$

where p is the channel detection probability.

***Lemma** 2:* Let Z_{ik} denote the event that passive node i selects channel k as its home channel using Algorithm 3. Then the probability of Z_{ik}, denoted as β, is given as

$$\beta = \Pr(Z_{ik}) = \tfrac{1}{M}\left(1 - (1-p)^{M}\right). \tag{3}$$

Note that β depends on M and p only, neither i nor k.

In each slot, an active node selects a passive node, estimates the home channel of this passive node, and then switches to the estimated home channel for packet transmission. We consider two strategies for an active node to select its intended passive node:

- *Random passive Node Selection* (RNS), i.e., randomly picking a passive node.
- *Traffic load-oriented passive Node Selection* (TNS). An active node j selects a passive node i if the active node has more packets to node i than to any other passive node in the current time slot.

The RNS can be seen as a special case of TNS in that, if the traffic load between all node pairs is uniform, then an active node has the same probability of selecting each passive node. Let ρ_{ji} denote the traffic load from node j to node i. Let $\mathcal{A}$ and $\mathcal{P}$ denote the set of active and passive nodes, respectively, in the current time slot. Let X_{ji} denote the event that active node j picks passive node i for packet transmission. Let η_{ji} denote $\Pr(X_{ji})$. η_{ji} is computed based on ρ_{ji}, as follows.

$$\eta_{ji} = \begin{cases} \frac{\rho_{ji}}{\sum_{i \in \mathcal{P}} \rho_{ji}}, \text{ for } j \in \mathcal{A},\ i \in \mathcal{P} \\ 0, \text{ otherwise} \end{cases} \tag{4}$$

Note that in the case of RNS, we simply have $\eta_{ji} = \frac{1}{|\mathcal{P}|}$ for $j \in \mathcal{A}$ and $i \in \mathcal{P}$.

Let Y_{ji} denote the event that active node j selects passive node i in the current time slot, and successfully switches to the home channel of node i. Then we have

$$\Pr(Y_{ji}) = \eta_{ji}\alpha.$$

In order to know the capacity for active node j to communicate with passive node i when they are at the same channel, we need to know the number of nodes in that channel. Next, we find the probability that there are $\tilde{n}$ passive nodes in the channel of nodes i and j, given that nodes i and j are on the same channel. Without loss of generality, let k denote the channel of nodes i and j. Let R denote the event that $\tilde{n}$ passive nodes are on channel k. Then we have

$$\begin{aligned} &\Pr(R \mid Y_{ji}) \\ &\quad = \binom{\tilde{N}-1}{\tilde{n}-1}\beta^{\tilde{n}-1}(1-\beta)^{\tilde{N}-\tilde{n}} \\ &\quad = B(\tilde{n}-1 : \tilde{N}-1, \beta), \end{aligned} \tag{5}$$

where $B(\tilde{n}-1 : \tilde{N}-1, \beta)$ denotes the Binomial distribution with the three parameters $\tilde{n}-1$, $\tilde{N}-1$, and β, respectively.

***Theorem** 1:* Let $\theta(\tilde{n})$ denote the probability that the home channel of the passive node selected by active node j' is channel k, given that there are $\tilde{n}$ passive nodes selecting channel k as their home channels. Then $\theta(\tilde{n}) = \frac{\tilde{n}}{\tilde{N}}$ for both RNS and TNS strategies of selecting passive nodes.

Proof: Let $\mathcal{F}_k$ denote the set of passive nodes that have

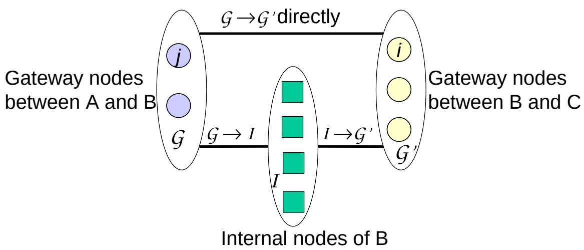

Fig. 2. Connections between gateway nodes of areas A, B, C, and internal nodes in area B.

selected channel k as the home channel. We have

$$\theta(\tilde{n}) = \sum_{\ell\in\mathcal{P}} \Pr(X_{j'\ell}, \ell \in \mathcal{F}_k) = \sum_{\ell\in\mathcal{P}} \Pr(X_{j'\ell}) \Pr(\ell \in \mathcal{F}_k)$$
$$= \sum_{\ell\in\mathcal{P}} \frac{\tilde{n}}{\tilde{N}} \eta_{j'\ell} = \frac{\tilde{n}}{\tilde{N}}.$$

Note that, in the above derivation, no matter $\eta_{j'i} = \frac{1}{|\mathcal{P}|}$ (for RNS) or as defined in (4), the final result is the same. In other words, $\theta(\tilde{n})$ does not depend on the strategy of selecting passive nodes. ■

Next, we analyze the probability that there are n active nodes switching to the channel of nodes j and i, denoted as channel k, given that there are $\tilde{n}$ passive nodes on channel k (including node i). Let U denote the event that n active nodes select the passive nodes on channel k and successfully switch to channel k. The probability of U is given as

$$\Pr(U \mid |\mathcal{F}_k| = \tilde{n}, Y_{ji}) = \binom{\bar{N}-1}{n-1} [\alpha\theta(\tilde{n})]^{n-1} (1-\alpha\theta(\tilde{n}))^{\bar{N}-n}$$
$$= B(n-1 : \bar{N}-1, \alpha\theta(\tilde{n})). \quad (6)$$

Let V denote the event that there are $\tilde{n}$ passive nodes and n active nodes on channel k, given that nodes j and i are both on channel k. Based on (5) and (6), we have

$$\begin{aligned}
&\Pr(V \mid Y_{ji}) \\
&\quad = \Pr(U, R \mid Y_{ji}) \\
&\quad = \Pr(U \mid R, Y_{ji}) \Pr(R \mid Y_{ji}) \\
&\quad = B(\tilde{n}-1 : \tilde{N}-1, \beta) B(n-1 : \bar{N}-1, \alpha\theta(\tilde{n})).
\end{aligned}$$

We define the capacity of a link or network as the *saturation throughput* of the link or network, which is defined as the limit reached by the system throughput as the offered load increases, and represents the maximum load that the system can carry in stable conditions. The reader is referred to [2] for an insightful discussion of saturation throughput.

Since we assume that the nodes on the same channel use an existing multi-access MAC protocol to access this channel for packet transmission, we adopt the analytical model in [2] to obtain the throughput on a channel, given the number of nodes on this channel. Let $T(n)$ denote the saturation throughput on a channel when there are n active nodes on this channel. Note that only active nodes transmit packets, while passive nodes receive packets.

On the channel of nodes j and i, i.e., channel k, when there are n active nodes and $\tilde{n}$ passive nodes, the capacity is $T(n)$, since we assume that only active nodes transmit in a time slot. We assume the MAC is a fair MAC, where each node has an approximately equal share of medium access opportunity. Furthermore, an active node communicates with its selected passive node only, and a passive node also communicates with the active node(s) that select it. Therefore, the capacity is equally shared by all active nodes, which is reasonable since we focus on saturation throughput. Hence, given that there are are n active nodes and $\tilde{n}$ passive nodes on channel k, the capacity between nodes j and i is $\frac{T(n)}{n}$. At last, we can get the mean capacity between nodes j and i, as follows:

$$\sum_{\tilde{n}=1}^{\tilde{N}} \sum_{n=1}^{\bar{N}} B(\tilde{n}-1 : \tilde{N}-1, \beta) B(n-1 : \bar{N}-1, \alpha\theta(\tilde{n})) \frac{T(n)}{n}.$$

VI. Forwarding Capacity of an Area

Next we consider the capacity between two areas across an intermediate area. We consider three areas, A, B, and C, where A and C are connected through area B. Let $\mathcal{G}$ denote the set of gateway nodes between A and B, and $\mathcal{G}'$ denote the set of gateway nodes between B and C. Let $\mathcal{I}$ denote the set of internal nodes of area B. Our objective is to find the capacity between the nodes in $\mathcal{G}$ and the nodes in $\mathcal{G}'$ across area B. To forward a packet from a gateway node in $\mathcal{G}$, say node j, to a gateway node in $\mathcal{G}'$, say node i, node j may directly send the packet to node i, or send it to an internal node and let the internal node forward the packet to node i. We illustrate the gateway nodes, internal nodes, and their connections in Fig. 2. Next, we need to find out how many active or passive nodes in $\mathcal{G}$, $\mathcal{G}'$, and $\mathcal{I}$, provided that their sizes, $|\mathcal{G}|$, $|\mathcal{G}'|$, and $|\mathcal{I}|$, are given, which are known from the information collected by the cluster and backbone nodes. Let N denote the number of nodes in area B. Note that N may be larger than $|\mathcal{G}|+|\mathcal{G}'|+|\mathcal{I}|$, since B may have other neighboring areas. Let $\tilde{N}$ and $\bar{N}$ denote the number of passive and active nodes, respectively, in the current time slot. By our algorithm, $\tilde{N} = \lceil \frac{N}{2} \rceil$ and $\bar{N} = \lfloor \frac{N}{2} \rfloor$. Since the node status is randomly selected, then given $\tilde{N}$ and $\bar{N}$, the probability of having n ($n \le |\mathcal{G}|$) active nodes is equivalent to the probability that among $|\mathcal{G}|$ balls drawn from an urn with $\bar{N}$ red balls and $\tilde{N}$ white balls, there are n red balls. Let $q(n : |\mathcal{G}|, \bar{N}, \tilde{N})$ denote this probability. Therefore, we have

$$q(n : |\mathcal{G}|, \bar{N}, \tilde{N}) = \frac{\binom{\bar{N}}{n}\binom{\tilde{N}}{|\mathcal{G}|-n}}{\binom{\bar{N}+\tilde{N}}{|\mathcal{G}|}}.$$

Similarly, the probability of having $\tilde{n}$ ($\tilde{n} \le |\mathcal{G}'|$) passive nodes in $\mathcal{G}'$ is

$$q(\tilde{n} : |\mathcal{G}'|, \tilde{N}, \bar{N}) = \frac{\binom{\tilde{N}}{\tilde{n}}\binom{\bar{N}}{|\mathcal{G}'|-\tilde{n}}}{\binom{\bar{N}+\tilde{N}}{|\mathcal{G}'|}}.$$

The probability of having h active nodes and $|\mathcal{I}|-h$ passive nodes in $\mathcal{I}$ is $q(h : |\mathcal{I}|, \bar{N}, \tilde{N})$. Given that there are n active nodes in $\mathcal{G}$ and $\tilde{n}$ passive nodes in $\mathcal{G}'$, there are $n\tilde{n}$ active-passive node pairs, and hence the aggregate capacity is $n\tilde{n}C(\tilde{N}, \bar{N})$. Furthermore, given that there are n active nodes in $\mathcal{G}$ and $|\mathcal{I}| - h$ passive nodes in $\mathcal{I}$, there are $n(|\mathcal{I}| - h)$

active-passive node pairs, and hence the aggregate capacity is $n(|\mathcal{I}| - h)C(\tilde{N}, \bar{N})$. Next, given that there are h active nodes in $\mathcal{I}$ and $\tilde{n}$ passive nodes in $\mathcal{G}'$, there are $h\tilde{n}$ active-passive node pairs, and hence the aggregate capacity is $h\tilde{n}C(\tilde{N}, \bar{N})$. The capacity from area A to C through the internal nodes in area B is

$$\min(n(|\mathcal{I}| - h)C(\tilde{N}, \bar{N}), h\tilde{n}C(\tilde{N}, \bar{N})).$$

At last, the total aggregate capacity from area A to area C is

$$n\tilde{n}C(\tilde{N}, \bar{N}) + \min(n(|\mathcal{I}| - h)C(\tilde{N}, \bar{N}), h\tilde{n}C(\tilde{N}, \bar{N})),$$

given that there are n active nodes in $\mathcal{G}$, $\tilde{n}$ passive nodes in $\mathcal{G}'$, and h active nodes in $\mathcal{I}$. Given the size of $\mathcal{G}$, $\mathcal{G}'$, $\mathcal{I}$, $\bar{N}$, and $\tilde{N}$ in area B, the forwarding capacity of area B for the traffic from area A to area C, denoted as $F(|\mathcal{G}|, |\mathcal{G}'|, |\mathcal{I}|, \bar{N}, \tilde{N})$, is given as

$$F(|\mathcal{G}|, |\mathcal{G}'|, |\mathcal{I}|, \bar{N}, \tilde{N}) = \sum_{n=0}^{|\mathcal{G}|} \sum_{\tilde{n}=0}^{|\mathcal{G}'|} \sum_{h=0}^{|\mathcal{I}|} q(n : |\mathcal{G}|, \bar{N}, \tilde{N}) \\ q(\tilde{n} : |\mathcal{G}'|, \tilde{N}, \bar{N}) q(h : |\mathcal{I}|, \bar{N}, \tilde{N}).$$

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[2] G. Bianchi, "Performance analysis of the ieee 802.11 distributed coordination function," *IEEE Journal on Selected Areas in Communications*, 2000.