Appendix

A. Proof of Theorem 1

Theorem 1: For two overlapped communities C_l, C_l' and an arbitrary relay set S ($S \subseteq C_l \cap C_{l'}$), we have:

$$B_{l,l'}(S) = \frac{1}{\sum_{v\in S}\lambda_{v,l}} + \frac{\sum_{v\in S}\lambda_{v,l}/\lambda_{v,l'}}{\sum_{v\in S}\lambda_{v,l}}.$$

Proof: According to Definitions 3 and 4, we have that $B_{l,l'}(S)$ is exactly the expected delivery delay from l to l' via the first encountered node in S. Since the time interval that each node v in S encounters l follows the exponential distribution with parameter $\lambda_{v,l}$, the probability density function of node v becoming the first node meeting l is $\lambda_{v,l}\prod_{v\in S} e^{-\lambda_{v,l}t}$. The delivery delay from l to l' via node v is t plus $D_{v,l'} = 1/\lambda_{v,l'}$. Then, we have:

$$\begin{aligned} B_{l,l'}(S) &= \sum_{v\in S}\Big(\int_0^\infty \lambda_{v,l}\prod_{v\in S} e^{-t\lambda_{v,l}}\big(t+1/\lambda_{v,l'}\big)dt\Big) \\ &= \frac{1}{\sum_{v\in S}\lambda_{v,l}} + \frac{\sum_{v\in S}\lambda_{v,l}/\lambda_{v,l'}}{\sum_{v\in S}\lambda_{v,l}}. \end{aligned} \tag{1}$$

□

B. Proof of Theorem 2

Theorem 2: Optimal Opportunistic Routing Rule: the message sender always delivers messages to the encountered relay that has a smaller minimum expected delay to the destination than itself. Concretely, a relay u belongs to the optimal relay set $\tilde{R}_i$ for the delivery from i to d, if and only if, $D_{u,d} < D_{i,d}$, i.e.:

$$u \in \tilde{R}_i \Longleftrightarrow D_{u,d} < D_{i,d} \tag{2}$$

Proof: We first prove $u \in \tilde{R}_i \Rightarrow D_{u,d} < D_{i,d}$ by contradiction. Assume that $u \in \tilde{R}_i$ while $D_{u,d} \ge D_{i,d}$. Then, we construct a new relay set $R^- = \tilde{R}_i - \{u\}$. By computing $D_{i,d}(\tilde{R}_i)$ and $D_{i,d}(R^-)$, we have:

$$\begin{aligned} D_{i,d}(\tilde{R}_i) &= \sum_{v\in\tilde{R}_i}\Big(\int_0^\infty \lambda_{i,v}\prod_{v\in\tilde{R}_i} e^{-t\lambda_{i,v}}\big(t+D_{v,d}\big)dt\Big) \\ &= \frac{1+\sum_{v\in\tilde{R}_i}\lambda_{i,v}D_{v,d}}{\sum_{v\in\tilde{R}_i}\lambda_{i,v}}, \end{aligned} \tag{3}$$

$$\begin{aligned} D_{i,d}(R^-) &= \sum_{v\in R^-}\Big(\int_0^\infty \lambda_{i,v}\prod_{v\in R^-} e^{-t\lambda_{i,v}}\big(t+D_{v,d}\big)dt\Big) \\ &= \frac{1+\sum_{v\in R^-}\lambda_{i,v}D_{v,d}}{\sum_{v\in R^-}\lambda_{i,v}}, \end{aligned} \tag{4}$$

Then, by comparing $D_{i,d}(\tilde{R}_i)$ and $D_{i,d}(R^-)$, we have:

$$D_{i,d}(\tilde{R}_i) - D_{i,d}(R^-) = \frac{\lambda_{i,u}}{\sum_{v\in R^-}\lambda_{i,v}}\big(D_{u,d} - D_{i,d}(\tilde{R}_i)\big). \tag{5}$$

That is:

$$D_{i,d}(\tilde{R}_i) \ge D_{i,d}(R^-) \Leftrightarrow D_{u,d} \ge D_{i,d}(\tilde{R}_i). \tag{6}$$

On the other hand, we have $D_{u,d} \ge D_{i,d} = D_{i,d}(\tilde{R}_i)$, according to the assumption. Thus, we can get $D_{i,d}(R^-) \le D_{i,d}(\tilde{R}_i)$ from Eq.(6). This is a contradiction in that $\tilde{R}_i$ is the optimal relay set to minimize $D_{i,d}$ (if there are multiple relay sets to minimize $D_{i,d}$, we always select the one with the smallest set size in this paper). Therefore, the assumption is wrong, and we should have $D_{u,d} < D_{i,d}$.

Likewise, we can get $D_{u,d} < D_{i,d} \Rightarrow u \in \tilde{R}_i$ by the contradiction method. Assume that $D_{u,d} < D_{i,d}$ and meanwhile $u \notin \tilde{R}_i$. Then, we construct a new relay set $R^+ = \tilde{R}_i + \{u\}$. By computing $D_{i,d}(R^+)$, we have:

$$\begin{aligned} D_{i,d}(R^+) &= \sum_{v\in R^+}\Big(\int_0^\infty \lambda_{i,v}\prod_{v\in R^+} e^{-t\lambda_{i,v}}\big(t+D_{v,d}\big)dt\Big) \\ &= \frac{1+\sum_{v\in R^+}\lambda_{i,v}D_{v,d}}{\sum_{v\in R^+}\lambda_{i,v}}. \end{aligned} \tag{7}$$

Then, by comparing $D_{i,d}(R^+)$ and $D_{i,d}(\tilde{R}_i)$ in Eq.(3), we have:

$$D_{i,d}(R^+) - D_{i,d}(\tilde{R}_i) = \frac{\lambda_{i,u}}{\sum_{v\in R^+}\lambda_{i,v}}\big(D_{u,d} - D_{i,d}(\tilde{R}_i)\big). \tag{8}$$

That is:

$$D_{i,d}(R^+) < D_{i,d}(\tilde{R}_i) \Leftrightarrow D_{u,d} < D_{i,d}(\tilde{R}_i). \tag{9}$$

On the other hand, we have $D_{u,d} < D_{i,d} = D_{i,d}(\tilde{R}_i)$ according to the assumption. Thus, we can get $D_{i,d}(R^+) < D_{i,d}(\tilde{R}_i)$ from Eq.(9). This is a contradiction in that $\tilde{R}_i$ is the optimal relay set to minimize $D_{i,d}$. Therefore, the assumption is wrong, and we should have $u \in \tilde{R}_i$. □

C. Proof of Theorem 3

Theorem 3: Assume that community C_l has m overlapped communities $C_{l_1}, \cdots, C_{l_m}$. Then, the optimal relay set $\tilde{R}_l$ of home l, and the optimal betweenness sets $\tilde{S}_{l,l_i}$ $(1 \le i \le m)$ satisfy:

1) if $v \notin \bigcup_{j=1}^m \tilde{S}_{l,l_i}$, then $v \notin \tilde{R}_l$;
2) $\tilde{S}_{l,l_i} \subseteq \tilde{R}_l$, otherwise $\tilde{S}_{l,l_i} \cap \tilde{R}_l = \emptyset$ for $\forall i \in [1,m]$.

Proof: 1. Since $v \notin \bigcup_{i=1}^m \tilde{S}_{l,l_i}$ means $v \in \bigcup_{i=1}^m \big(C_l \cap C_{l_i} - \tilde{S}_{l,l_i}\big)$, then without loss of generality, we assume $v \in C_l \cap C_{l_i} - \tilde{S}_{l,l_i}$ and $v \in \tilde{R}_l$ to prove the first property by contradiction. Firstly, we construct a new relay set R^- for the message delivery from l to d via $l_1, \cdots, l_m$. Let $R^- = \tilde{R}_l - (C_l \cap C_{l_i}) + \tilde{S}_{l,l_i}$, and then compare the delay values, $D_{l,d}(R^-)$ and $D_{l,d}(\tilde{R}_i)$, the delivery delays from l to d via the new relay set R^- and the optimal relay set $\tilde{R}_l$. In fact, the two delay values are the expected values of the delays via nodes in the two relay sets. Consider that a node in $R = \tilde{R}_l - (C_l \cap C_{l_i})$ first visits l and is selected as the real relay. Its contributions to $D_{l,d}(R^-)$ and $D_{l,d}(\tilde{S}_{l,l_i})$ are the same. Thus, we only need to consider the contributions of the remaining nodes in $R^- - R$ $(= \tilde{S}_{l,l_i})$ and $\tilde{R}_l - R$ to $D_{l,d}(R^-)$ and $D_{l,d}(\tilde{R}_l)$, respectively. Since $\tilde{S}_{l,l_i}$ is the optimal relay set for the direct delivery from l to l_i, we thus have $D_{l,l_i}(\tilde{S}_{l,l_i}) + D_{l_i,d} < D_{l,l_i}(\tilde{R}_l - R) + D_{l_i,d}$. That is, the expected delay from l to l' via R^- is even less than the delay via $\tilde{R}_l$. This is a contradiction in that $\tilde{R}_l$ is the optimal relay set. Therefore, the assumption about $v \in \tilde{R}_l$ is wrong, and we should have $v \notin \tilde{R}_l$.

2. We are still using the contradiction method, and assume that there exists an integer $i \in [1, m]$ that satisfies $\tilde{S}_{l,l_i} \not\subseteq \tilde{R}_l$ and $\tilde{S}_{l,l_i} \cap \tilde{R}_l = R \neq \emptyset$. We also construct a new relay set $R' = \tilde{R}_l - R + \tilde{S}_{l,l_i}$. Based on a similar analysis as in part 1, we have that $D_{l,d}(R')$ is less than $D_{l,d}(\tilde{R}_l)$. This is a contradiction in that $\tilde{R}_l$ is the optimal relay set. Therefore, the assumption about $\tilde{S}_{l,l_i} \cap \tilde{R}_l \neq \emptyset$ is wrong, and the theorem is correct. □

D. Proof of Corollary 2

Corollary 1: CAOR can achieve the minimum expected delivery delay.

Proof: A straightforward result in Section 4.3. □

Corollary 2: Assume that $\lambda_{v_1,l'} \geq \lambda_{v_2,l'} \geq \cdots \geq \lambda_{v_n,l'}$, then the optimal betweenness set $\tilde{S}_{l,l'}$ satisfies:

1) $v_1 \in \tilde{S}_{l,l'}$;
2) if $v_{i+1} \in \tilde{S}_{l,l'}$, then $v_i \in \tilde{S}_{l,l'}$. That is, $\exists k \in [1, n]$ s.t. $\tilde{S}_{l,l'} = \{v_1, \cdots, v_k\}$;
3) if $\tilde{S}_{l,l'} = \{v_1, \cdots, v_k\}$, then $B_{l,l'}(\{v_1, \cdots, v_i\}) > B_{l,l'}(\{v_1, \cdots, v_i, v_{i+1}\})$ for any $i \in [1, k-1]$.

Proof: At first, we directly prove the second result, which also implies the first result. We consider the optimal opportunistic routing between l and l' via $\{v_1, \cdots, v_n\}$. if $v_{i+1} \in \tilde{S}_{l,l'}$, then we have $D_{v_{i+1},l'} < D_{l,l'}$ according to Theorem 2. Since $D_{v_i,l'} = \frac{1}{\lambda_{v_i,l'}} < D_{v_{i+1},l'} = \frac{1}{\lambda_{v_{i+1},l'}}$, we can get $D_{v_i,l'} < D_{l,l'}$. Using Theorem 2 again, we have $v_i \in \tilde{S}_{l,l'}$. Without loss of generality, let the node in $\tilde{S}_{l,l'}$ with the largest expected delay to community home l' be v_k, i.e., $v_k \in \tilde{S}_{l,l'}$. Then, $v_{k-1}, v_{k-2}, \cdots, v_1 \in \tilde{S}_{l,l'}$, i.e., $\tilde{S}_{l,l'} = \{v_1, \cdots, v_k\}$.

Now we prove the third result. Compare $D_{l,l'}(\{v_1, \cdots, v_i\})$ and $D_{l,l'}(\{v_1, \cdots, v_i, v_{i+1}\})$, we have:

$$\begin{aligned} D_{l,l'}(\{v_1, \cdots, v_{i+1}\}) < D_{l,l'}(\{v_1, \cdots, v_i\}) \Leftrightarrow \\ D_{v_{i+1},l'} < D_{l,l'}(\{v_1, \cdots, v_i\}). \end{aligned} \tag{10}$$

On the other hand, $v_{i+1} \in \tilde{S}_{l,l'}$, then we can get $D_{v_{i+1},l'} < D_{l,l'} < D_{l,l'}(\{v_1, \cdots, v_i\})$ according to Theorem 2. Thus, $D_{l,l'}(\{v_1, \cdots, v_{i+1}\}) < D_{l,l'}(\{v_1, \cdots, v_i\})$. □