Finding Master Nodes of Hopfield Networks using Non-Axiomatic Logic

Tangrui Li[1 [0009−0005−3471−038X]]

Temple University, Philadelphia PA 19122, USA {tuo90515}@temple.edu

Abstract. In the Hopfield network, there are neurons that are more useful for pattern reconstruction (master nodes), but the existed work of finding master nodes relies on destroying the network topological structure, thus damaging the semantics of Hopfield networks. This paper discusses the semantic similarity between Hopfield networks and Non-Axiomatic Logic (NAL) and solves the problem of finding master nodes in Non-Axiomatic Reasoning System (NARS) by representing Hopfield networks as Narsese sentences. Through experiments, our proposed method can reduce the pattern reconstruction error by up to 28%.

1 Introduction

The Hopfield network is a network that stores and reconstructs patterns through binarized neurons. Each neuron is connected to all other neurons in the network. This connection is often bidirectional and has a weight, where the weight can be thought of as the strength of the influence of two neurons on each other. Each neuron of the network can be in one of two states: active or inactive, usually represented by +1 and -1. In actual use, there may also be a default state, usually represented by 0.

Hopfield networks are based on some energy functions, which are determined by the connection weight and the activation state of the neuron. The purpose of the network is to minimize the overall energy by updating the state of the neuron given learned weights, so that the network can converge to a previously stored pattern. In a traditional Hopfield network, the number of attractors is approximately 15% of the number of neurons. In this paper what called model Hopfield networks [1] will not be discussed, since they sacrifice the interpretability and semantics in exchange of representative power.

Hebbian Learning is a learning and state updating mechanism applied in Hopfield networks. It is often summarized as "neurons that are activated and deactivated together will strengthen their connections with each other." When a state defaults, the weight connected to it will not take effect (0 times any number is 0), so even if two states default at the same time, the weight between them will not increase.

Please consider "sweet sugar". The description of the sweetness is redundant. This is because in our cognition, when "something is sugar" is true, "something is sweet" is often also true. True, we can describe the behavior of the Hopfield network as a part of NAL. Suppose a pattern consists multiple states (e.g., sugar, sweet). For the network, each state is abstracted as a neuron. When the state is true, the neuron is activated, otherwise it is deactivated. The purpose of the network is to obtain the mutual activation relationship among all neurons. Described in the way of NAL, each pattern can be described as a combination of multiple judgments, and the purpose of the network in NAL is to obtain the equivalence between these judgments.

2 Hopfield Networks on Pattern Reconstruction

After the Hopfield network completes the memorization of the patterns, if a pattern is polluted in a certain proportion (flips or sets it to 0), and the network can still restore the pattern to some extent (concerning different pollution proportions). The following experiment is used to illustrate this point, assuming a Hopfield network with 36 neurons and a total of 100 random patterns memorized. When these patterns are flipped or set to 0 at a certain percentage, [Fig. 1, 2] depicts the mean squared error between the pattern reconstructed and the original pattern. Obviously, the performance is good when the pollution ratio is low.

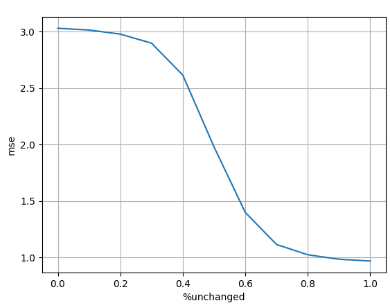


Fig. 1: Error and the unpolluted proportion when flipping is applied.

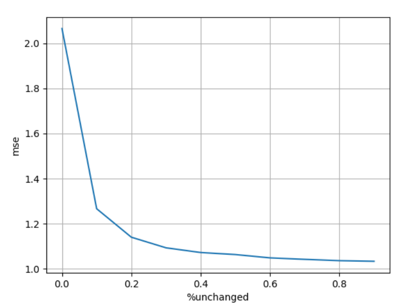


Fig. 2: Error and the unpolluted proportion when defaults are applied.

But taking the pollution rate 40% as an example, I conducted 10 experiments, randomly selecting neurons for destruction each time, and I got the error as shown in the [Fig. 3, 4]. It can be seen that the performance is not average, which shows that in the network, there are neurons with stronger reconstruction ability than other neurons (master nodes). When these nodes are not polluted, the error is smaller.

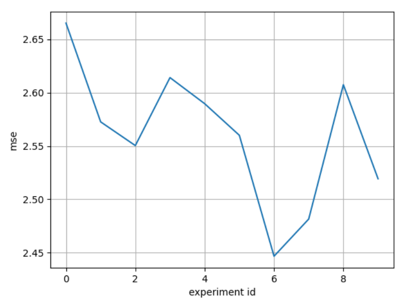


Fig. 3: Errors in different experiments when flipping is applied.

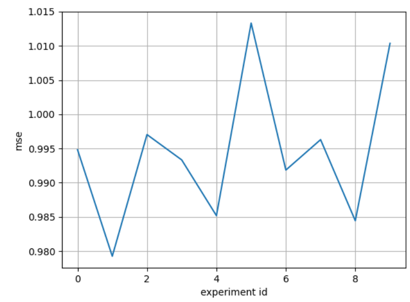


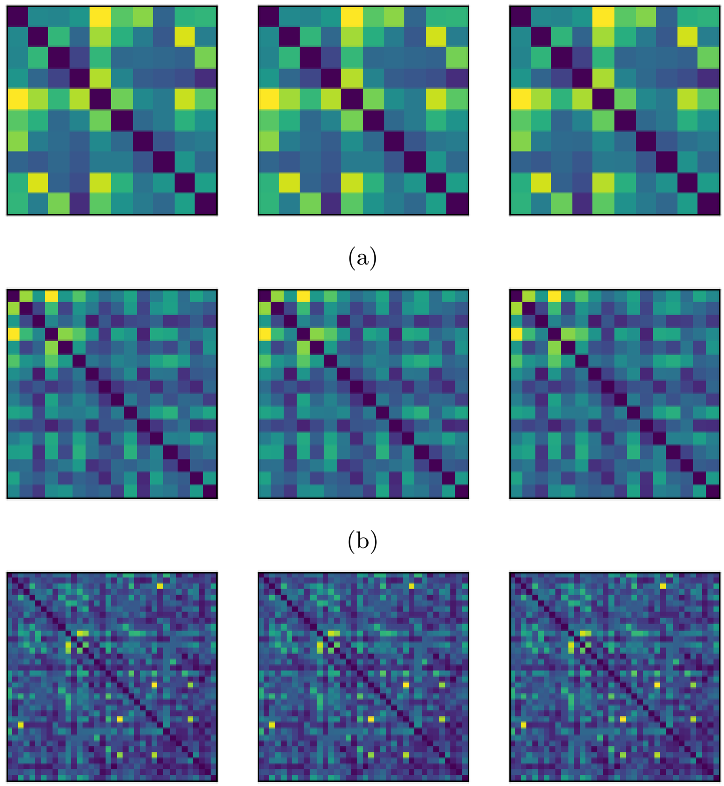
Fig. 4: Errors in different experiments when defaults are applied.

3 Method

There are very few existing studies that discuss the problem of master node in Hopfield network, but this research [2] is by changing the topology of the neural network (making it from a fully connected undirected graph to a selectively connected directed graph), so it cannot explain the above experimental phenomenon. I chose to use the semantic similarity between Hopfield Networks and NAL to solve this problem in NARS.

For any two states (or judgments in NARS) $A. < f, c >$ and $B. < f, c >$, if they are true at the same time, we translate it as $A \leftrightarrow B. < 1; 0.5 >$, otherwise the frequency is 0. A confidence of 0.5 means that each data sample contributes 1 unit of evidence. I chose to treat the equivalence between states as non-events. Correspondingly, the states themselves are treated as events (e.g., $A. :|:$ and $B. :|:$), so that the truth values of them expire with the system cycles and thus without being affected by the previous data item. After reading a data sample and reasoning for a while, how NARS sees the mutual activation of these states is recognized by asking NARS questions for the truth values of all connections (e.g., $A \leftrightarrow B?$).

Assuming there are n neurons, there is an equivalence of $O(n^2)$ for each data item. Therefore, I chose to process data in batches. If a single data item contributes 1 unit of evidence, then a batch contains multiple pieces of evidence, and its truth value is no longer a Boolean value. On this basis, I chose equivalences with more extreme truth values (>0.8 or <0.2) into NARS to greatly reduce the amount of Narsese. In addition, we use fewer inference cycles on each data sample and use a higher number of training sample repetitions to speed up the system's response. After reading each data sample, it does not wait for NARS to answer all questions. Instead, it specifies a fixed waiting time window to record the output conclusions. We accumulate them over multiple cycles, roughly describing the overall strength of connection between states as shown in [Fig. 5].



(a)

(b)

(c)

Fig. 5: Expectation (left), frequency (mid.) and confidence (right) of equivalences among judgments when different number of states are used.

I sorted the states according to their average connection strength, thereby giving priority to higher ones. Compared with the reconstruction performance of randomly selected states, [Fig. 6] is obtained. I used an example of 10 states and 100 patterns to discuss the difference between the method proposed and the random selection of retained neurons at different pollution ratios.

4 Conclusion

Through the semantic similarity between Hopfield networks and NAL, this paper proposes a method to find master nodes and verifies the effectiveness of these nodes in pattern reconstruction. However, the complexity of the data used in this paper is not high enough. In future work, more energy will be placed on further application of the assumption of insufficient knowledge and resources (AIKR).

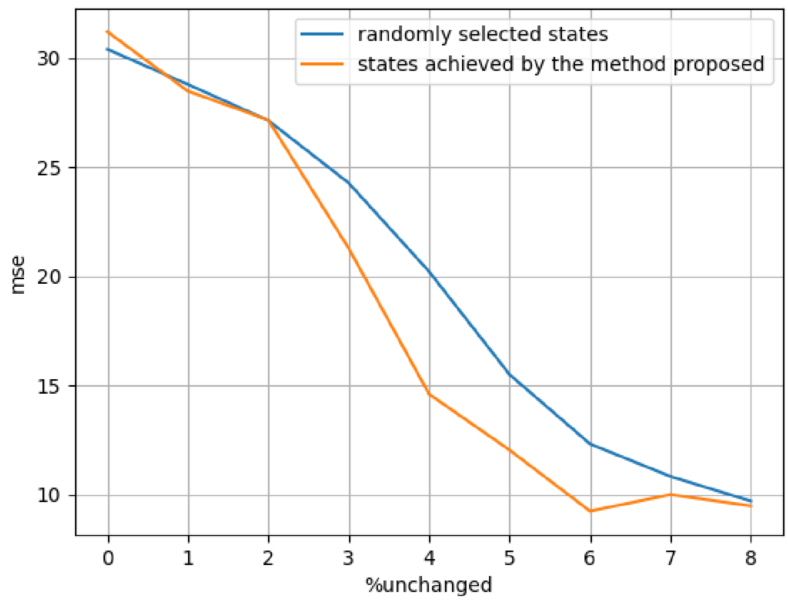


Fig. 6: Errors from retaining the randomly selected states and states selected by the proposed in different pollution rates.

References

1. Ramsauer H, Schäfl B, Lehner J, Seidl P, Widrich M, Adler T, Gruber L, Holzleitner M, Pavlović M, Sandve GK, Greiff V. Hopfield networks is all you need. arXiv preprint arXiv:2008.02217. 2020 Jul 16.
2. Rodgers N, Tiňo P, Johnson S. Network hierarchy and pattern recovery in directed sparse Hopfield networks. Physical Review E. 2022 Jun 13;105(6):064304.