Making 3D binary digital images well composed

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A three-dimensional binary digital image is said to be well composed if, and only if, the set of points in the voxel faces consisting of every voxel face that is shared by a foreground and a background voxel of the image is a 2D manifold.[1] A well-composed image enjoys very useful topological and geometric properties. These properties make simpler several basic algorithms in computer vision, computer graphics, and image processing. For instance, thinning algorithms do not suffer from the irreducible thickness problem if the image is well composed.[2] Also, algorithms that rely on curvature computation to extract approximating iso-surfaces directly from binary images can be applied to well-composed images with no need to handle special cases resulting from 'non-manifold' topology.[3,4]

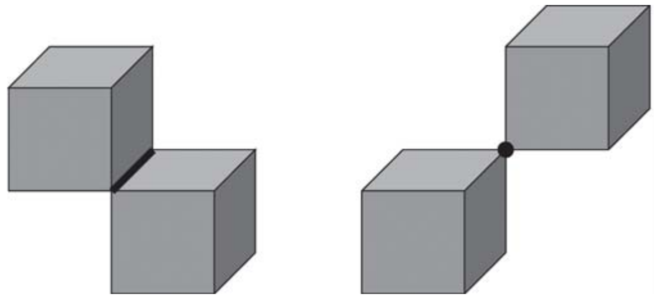

Figure 1. Instances of the two critical configurations. For the sake of clarity, only the background voxels are shown. The shared edge (left) of the first critical configuration and the shared vertex (right) of the second are heavily drawn.

On the other hand, if a 3D digital binary image is the result of the digitization of a 'solid' object, such as a bone, and it lacks the property of being well composed. In this case, the digitization process that gave rise to it is not topology-preserving. As the results in Reference 5 show, if the resolution of the digitization process is fine enough to ensure preservation of topology, then the resulting image is well composed. This fact has motivated us to develop an iterative and randomized algorithm for 'repairing' non well-composed 3D digital binary images.

Our algorithm restores the given image by converting background voxels into foreground ones. Although this algorithm always produces a well-composed image, it cannot guarantee that the result is the same as would be obtained from a topology-preserving digitization process. This is because our algorithm does not assume any knowledge about the original digitization process. Even so, if the number of background voxels converted into foreground ones is not too large, the input and output images will be similar, which is satisfactory for several applications that benefit from using well-composed images.

The conversion process relies on the fact that well composedness is a local property: that is, the condition of being well composed is equivalent to the nonexistence of two types of local critical configurations of image voxels[1] (see Figure 1). The first of these is where four voxels share an edge, two of them are background voxels, two of them are foreground voxels, and the background (or foreground) voxels share an edge but not a face. The second is where eight voxels share a vertex, two of them are background (or foreground), six of them are foreground (or background) voxels, and the two background (or foreground) voxels share a vertex but not an edge. It can be shown that a 3D binary digital image is well composed if, and only if, it does not contain any instances of these critical configurations.

Note that we can decide if a given 3D binary digital image is well composed by simply verifying if any 2×2×1 neighborhood of voxels of the image is an instance of the first critical configuration, and if any 2×2×2 neighborhood of voxels of the image is an instance of the second. This test can be performed in linear time with the number of voxels of the image. The first step of our algorithm is to verify if the input 3D binary digital image is well composed. If so, the algorithm finds a subset P of background voxels such that, if they were converted into foreground voxels, then the resulting image would be well composed. Then this conversion is performed.

Ideally, P should be as small as possible, so that the input and output images are similar. Although such a smallest set can be found using an exponential-time search, this is completely unfeasible in the context of practical applications. Our repairing algorithm is not guaranteed to find the smallest set P, but its time complexity is linear in the number of voxels of the input image. Our algorithm builds P iteratively, starting with an empty set. Each iteration inserts a background voxel into P. Each such voxel is randomly chosen from those in the background in the critical configurations of the input image, and when such a voxel is converted into a foreground one, at least one instance of a critical configuration is eliminated. This conversion can also give rise to a new critical configuration, which is further eliminated by choosing another background point. However, this process is guaranteed to converge to a correct solution after a finite number of iterations.

We tested our algorithm against several magnetic resonance (MR) images of parts of the human body, such as the brain, torso, and lungs. In all cases, our algorithm generated a well-composed image by converting fewer than 0.0034*N, where N is the total number of voxels in the input 3D binary digital image. We also derived an upper bound for the average number of voxels that needed to be converted. This upper bound supports the results obtained from our experiments with real imaging data, and provides a theoretical measure of the effectiveness of our algorithm for making 3D binary digital images well-composed. For future work, we intend to study the existence of a linear-time algorithm for computing the smallest subset of background voxels that generates a well composed image after being converted into foreground voxels.

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