APPENDIX

Theorems 1 and 2 proven in this section follow from the theoretical properties of the skeleton that in [32]. In particular, we make a heavy use of the Domain Decomposition Lemma (Theorem 5.1 in [32]).

Theorem 1 *Let* $[x, y]$ *be a contour segment that belongs to some contour partition* Γ *of* ∂D*. If* D *is simply connected,(i.e.,* ∂D *consists of only one simple closed curve), then* $S(D) - S(CS((x, y)))$ *is a strong deformation retract of* $S(D)$ *(i.e.,* $S(CS((x, y)))$ *can be removed from the skeleton* $S(D)$ *without violating its topology*).

Proof: We assume that there exists a skeleton point all of whose generating points lie on (x, y). If this is not the case, then the theorem is true, since then $CS((x, y))= \varnothing$.

Let $C = \partial D$ be the only boundary curve of D. Clearly, we have $[x, y] \subset C$.

It follows from Theorem 5.1 in [32] that there exists a point $s \in S(D)$ such that $C_i(s) \subset [x, y]$, where C_i is one of the contour components $C_1(s), \ldots, C_k(s)$ of $\partial D - B(s)$, and for any other point $p \in S(D)$, whenever $C_j(p) \subset [x, y]$ for some component $C_j(p)$ of $\partial D - B(p)$, we have $C_j(p) \subset C_i(s)$.

We will show that $C_i(s) = CS((x, y))$. Clearly $CS((x, y)) \subset C_i(s)$.

It remains to show that $C_i(s) \subset CS((x, y))$.

Let $x \in C_i(s)$. Then $S(x) \in S_i(s)$ and consequently $S^{-1}(S(x)) \subset C_i(s)$. Since $C_i(s) \subset (x, y)$, we obtain that $x \in CS((x, y))$.

Now we are ready to construct the strong deformation retraction f. The key observation is that f maps $S(D)$ to $S(D) - S(CS((x, y)))$ by simply mapping $S(CS([x, y]))$ to the point s.

By Theorem 8.1 in [32], S: $\partial D \rightarrow S(D)$ is a strong deformation retraction. This implies that $S(CS([x, y])) = S_i(s) \cup \{s\}$ is a strong deformation retract of $CS([x, y])$, and consequently, $\{s\}$ is homotopy equivalent to $S_i(s) \cup \{s\}$. Therefore, it is possible to define a strong deformation retraction f that maps $S(CS([x, y]))$ to $\{s\}$, and is identity on $S(D) - S(CS([x, y]))$. This proves the theorem.

Theorem 2 *Let* $[x, y]$ *be a contour segment that belongs to some contour partition* Γ *of* ∂D*. If* $CS((x, y))$ *is a subsegment of* (x, y) *(i.e.,* $CS((x, y))$ *is arc connected), then* $S(D) - S(CS((x, y)))$ *is a strong deformation retract of* $S(D)$*.*

Proof: We assume that there exists a skeleton point whose all generating points lie on (x, y). If this is not the case, then the theorem is true, since then $CS((x, y)) = \emptyset$.

Let $C \subset \partial D$ be a boundary curve such that $[x, y] \subset C$.

Our first step is to show that there exists some point $s \in S(D)$ and two contour points $a, b \in C$ such that $CS([x, y]) = [a, b] \subset [x, y]$, where $[a, b]$ is the shortest subsegment of $[x, y]$ that contains points a and b, see Fig. 8.

In order to prove this fact, we will show that $CS([x, y])$ is equal to the union of all arcs $arc(S(z), [x, y])$ induced by skeleton points $S(z)$ whose all gene rating points are contained in the contour segment $[x, y]$, (i.e., $CS([x,y]) = \bigcup \{arc(S(z),[x,y]) : S^{-1}(S(z)) \subset [x,y]\}$).

We have $z \in CS([x, y])$ iff $S^{-1}(S(z)) \subset [x, y]$, and since $CS([x, y])$ is arc connected, $S^{-1}(S(z)) \subset [x, y]$ iff $arc(S(z), [x, y]) \subset CS([x, y])$.

It follows from Theorem 5.1 in [32], that for any two points $u, v \in CS([x, y])$, we have

$arc(S(u), [x, y]) \subset arc(S(v), [x, y])$ or $arc(S(v), [x, y]) \subset arc(S(u), [x, y])$.

Therefore, $CS([x, y])$ is the union of an increasing sequence of closed contour segments, and consequently, $CS([x, y])$ is a closed contour segment, (i.e., $CS([x, y]) = [a, b] \subset [x, y]$ for some a, b $\in C$). Since the points a and b are the limits of endpoints of segments arc($S(z)$, $[x, y]$) such that $S^{-1}(S(z)) \subset [x, y]$ and such that the two endpoints of each arc map to the same skeleton point, we obtain that such that S(a)=S(b)=s for some point $s \in S(D)$, see Fig.8.

Now we are ready to construct the strong deformation retraction f. The construction is very similar to the one in the proof of Theorem 1. First observe that f maps $S(D)$ to $S(D) - S(CS((x, y)))$ by simply mapping $S(CS([x, y]))$ to the point s.

By Theorem 8.1 in [32], $S\colon \partial D \rightarrow S(D)$ is a strong deformation retraction. This implies that $S(CS([x, y])) = S([a, b]) = S(arc(s, [x, y]))$ is a strong deformation retract of $[a, b]$. Since $[a, b]$ has two endpoints in C that are both mapped by S to the same skeleton point s, $S([a, b]) = S(CS([x, y]))$ has exactly one endpoint s in $S(D)$, and $\{s\}$ is homotopy equivalent to $S([a, b])$. Therefore, it is possible to define a strong deformation retraction f that maps $S([a, b])$ to $\{s\}$, and is identity on $S(D) - S(CS((x, y)))$. This proves the theorem.

Theorem 3 *Let v be a vertex in* P^{n-k} *but not in* $P^{n-(k+1)}$ (*removed by DCE*) *or v be convex in* P^{n-k} *and concave in* $P^{n-(k+1)}$*. If there is a skeleton branch that ends at v in the skeleton pruned with partition induced by* P^{n-k}*, then exactly this branch is removed to obtain the skeleton pruned with partition induced by* $P^{n-(k+1)}$.

Proof: We only prove this theorem in the case of the removal of vertex v. The proof in the case of the status change form convex to concave is analogous.

Fig.19 shows part of polygon with several vertices of which v_0, v_1, v_2 are convex and v_3 is concave. (Thick segments represent skeleton branches inside the polygon.) Each convex vertex can be seen as a disk with a radius of zero, which is an endpoint of a skeleton branch engendered by its two adjacent line segments.

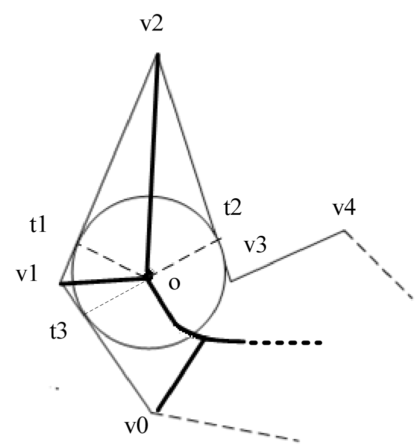

Figure 19. When vertex $v=v_2$ is removed from the contour partition, the branch ov_2 ending at it, is completely removed by the proposed skeleton pruning.

Observe that a maximal disk cannot have more than one tangent point on each line segment of the contour polygon. For example, in Fig.19, the maximal disc of the junction point o has three tangent points t_1, t_2 and t_3 on three different boundary segments v_0v_1, v_1v_2, and v_2v_3, respectively. It follows from Theorem 5.1 in [32] that the skeleton branch ov_2 must be equal to one of the $S_i(o)$ for i=1,2,3, say it is $S_1(o)$. Then the boundary segment (t_1, t_2) is equal to $C_1(o)$. Consequently, all the skeleton points on $S_1(o)$ have their generating points on $C_1(o)$. Therefore, when $v=v_2$ is deleted by DCE, the whole branch $ov_2=S_1(o)$ is removed from the skeleton. However, the junction point o remains, for two reasons. First, because we prune with respect to open contour segment (t_1, t_2), and second it has another tangent point t_3 that does not belong to $[t_1, t_2]$. This proves the theorem.