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# SKELETON-BASED SHAPE CLASSIFICATION USING PATH SIMILARITY

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Most of the traditional methods for shape classification are based on contour. They often encounter difficulties when dealing with classes that have large nonlinear variability, especially when the variability is structural or due to articulation. It is well-known that shape representation based on skeletons is superior to contour based representation in such situations. However, approaches to shape similarity based on skeletons suffer from the instability of skeletons, and matching of skeleton graphs is still an open problem.

Using a new skeleton pruning method, we are able to obtain stable pruned skeletons even in the presence of significant contour distortions. We also propose a new method for matching of skeleton graphs. In contrast to most existing methods, it does not require converting of skeleton graphs to trees and it does not require any graph editing. Shape classification is done with Bayesian classifier. We present excellent classification results for complete shapes.

*Keywords*: Shape classification; skeletons; shape similarity; skeleton pruning; Bayesian classifier.

## 1. Introduction

An important goal in image analysis is to classify and recognize objects. They can be characterized in several ways, using color, texture, shape, movement and location. Shape, as a significant factor of objects, is an important research direction in image classification and recognition. Shape of planar objects can be described based on their contours or on skeletons.

When utilizing contours in classification and recognition, shape classes that have a large nonlinear variability of global shape, due to structural variation, articulation or other factors, present a challenge for several existing shape recognition approaches. Approaches that match the target shape to stored example shapes require a large number of stored examples to capture the range of variability.[9]

Furthermore, existing example- and model-based approaches cannot handle object classes that have different parts or numbers of parts without splitting the class into separate subclasses. This type of structural variation can be handled by approaches that represent part relationships explicitly and match shapes syntactically; however, these structural approaches are computationally expensive.[23]

On the other hand, skeleton (or medial axis), which integrates geometrical and topological features of the object, is an important shape descriptor for object recognition.[10] Shape similarity based on skeleton matching usually performs better than contour or other shape descriptors in the presence of partial occlusion and articulation of parts.[8,9,17,22] There exists a large number of algorithms to compute skeletons. We only review some of them. Ablameyko *et al.*[1] presented an algorithm to construct the hierarchical structure graph of the object by decomposition of the distance-labeled skeleton into its meaningful structure elements. This graph can exactly describe the topological relationship of its structure elements. Borgefors *et al.*[11] introduced a procedure to hierarchically decompose a multiscale discrete skeleton. The method led to good performance on patterns having different thickness in different regions. Several methods related to skeleton computation have been introduced and analyzed by Arcelli and Sanniti di Baja.[2] Though the existing method on skeleton shows good performance in some cases, it is still a challenging task to automatically recognize the objects using their skeletons due to skeleton sensitivity to boundary deformation.[25] Usually the skeleton branches have to be pruned for recognition.[4,6,12,20,25,29] Moreover, another major restriction of recognition methods based on skeleton is a complex structure of obtained tree or graph representations of the skeletons. Graph edit operations are applied to the tree or graph structures, such as merge and cut operations,[13,15,19,21,30] in the course of the matching process. Probably the most important challenge for skeleton similarity is the fact that the topological structure of skeleton trees or graphs of similar object may be completely different. Besides, some methods[27] have focused on utilizing geometry measure to gauge the similarity of 2D shapes by comparing their skeletons. This fact is illustrated in Fig. 1. Although the skeletons of the two horses (a) and (b) are similar, their skeleton graphs (c) and (d) are very different. This example illustrates the difficulties faced by approaches based on graph edit operations in the context of skeleton matching. To match skeleton graphs or skeleton trees like the ones shown in Fig. 1, some nontrivial edit operations (cut, merge, etc.) are inevitable. On the other hand, skeleton graphs of different objects may have the same topology as shown in Fig. 2. The skeletons of the brush in Fig. 2(a) and the pliers in Fig. 2(b) have the same topology as shown in Fig. 2(c).

The proposed method combines Bayesian classifier and a novel skeleton representation that overcomes the above limitations. This paper utilizes a three-level statistical framework including distinct models for database, class and part. Bayesian inference is used to perform classification within this framework. Based on Bayes rule, the posterior probabilities of classes can be computed by the difference between skeletons of query shape and the shape in database. In the proposed framework, it

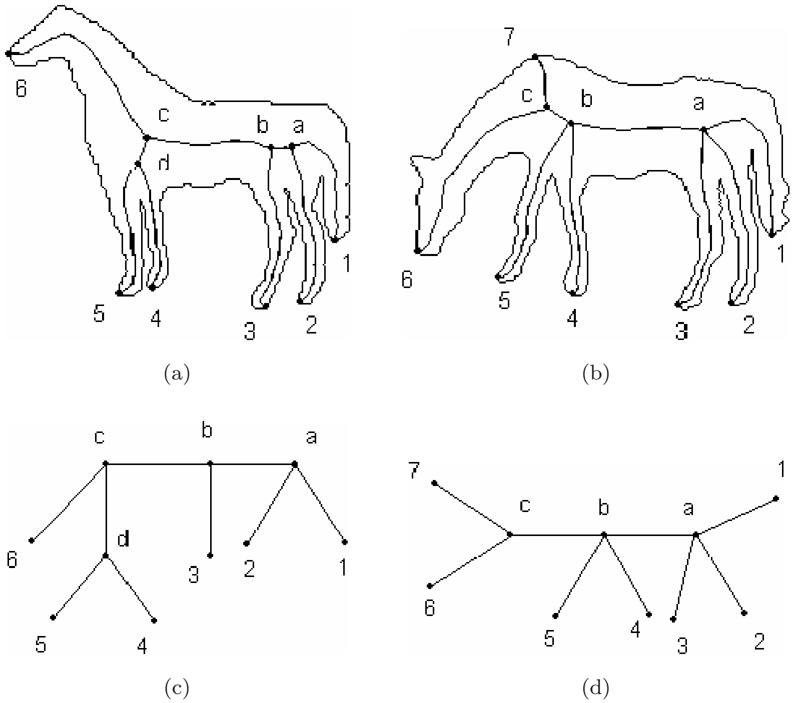



Fig. 1. Visually similar shapes in (a) and (b) have very different skeleton graphs in (c) and (d).

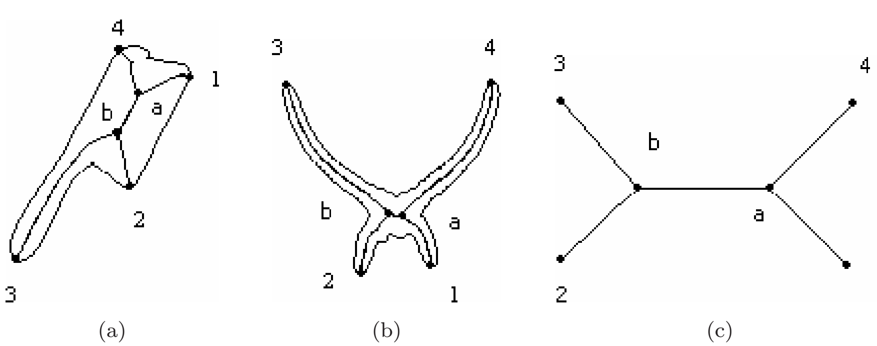



Fig. 2. Dissimilar shapes in (a) and (b) can have the same skeleton graphs (c).

can work well to classify complete shapes. The outline of the proposed method is as follows:

(1) Stable skeletons are obtained by the skeleton pruning method,[6] which is briefly described in Sec. 3.
(2) The probability of two different paths being similar is obtained as Gaussian of the distance between them. The definition of skeleton path is presented in Sec. 4 and the way to calculate the probability is described in Secs. 4 and 5.

(3) In Sec. 5, we compute the probability that a given shape, usually composed of several skeleton paths, belongs to a given shape class. The computation is based on Bayesian classifier and the probabilities of pairs of paths being similar.

## 2. Relevant Work on Shape Matching

This section briefly introduces some recent methods developed for shape matching, including classification, detection and retrieval.

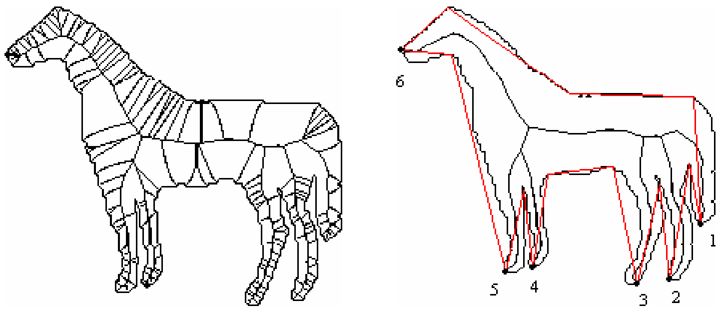
A number of approaches are based on the contour. Belongie *et al.*[9] proposed the concept of "shape context", which are log-polar histograms among different points on the shape. Through finding the correspondence between points on different shapes, this approach can obtain the similarity between the shapes. Some methods used boosting to classify objects. Bar-Hillel *et al.*[7] designed a classifier based on a part-based, generative object model. The approach given by Opelt *et al.*[14] developed a novel learning algorithm which uses Adaboost to learn jointly based on shape features. Besides the learning algorithm, Gorelick *et al.*[16] used Poisson equation to extract various shape properties for shape classification. Sun and Super[26] used distribution of contour parts in known object classes to classify shapes with Bayesian classifier. Their classification works only for complete query shapes.

In contrast to the methods based on contour, many researchers have worked on the approaches based on skeleton. Zhu *et al.* matched the skeleton graphs of objects using a branch-bounding method that is limited to motionless objects.[30] Shock graph is a kind of ARG proposed by Siddiqi *et al.*, which is based on the shock Grammar. The distance between subgraphs is measured by comparing the eigenvalues of their adjacency matrices. Though there are a lot of methods for shape similarity based on skeleton, few approaches implement the skeleton in classification. The main reason for this is that the past methods have high complexity. The proposed method defines a novel approach to classify the shape.

## 3. Skeleton Pruning

Any topology preserving method can be used to compute skeletons. We used the method by Choi *et al.*[12] The limitation of skeleton is that it is sensitive to the boundary deformation and the noise. Therefore, it is difficult to obtain the ideal skeletons to recognize the objects. In order to solve this problem, this method utilizes skeleton pruning introduced in Ref. 6 to improve the skeleton. First, Discrete Curve Evolution (DCE) simplifies the polygon. Then the skeleton is pruned so that only branches ending at the convex DCE vertices remain. For example, in Fig. 3(a), the skeleton contains a lot of noise. In Fig. 3(b), all the endpoints (denoted by $1, 2, \ldots, 6$) of the horse's skeleton are vertices of the DCE simplified polygon (in red). The pruned skeleton is guaranteed to preserve the topology of the shape and it is robust to noise and boundary deformation.[6]

The main benefit of using DCE is the fact that DCE is context sensitive. It recursively removes least relevant polygon vertices, where the relevance measure is




(a) (b)

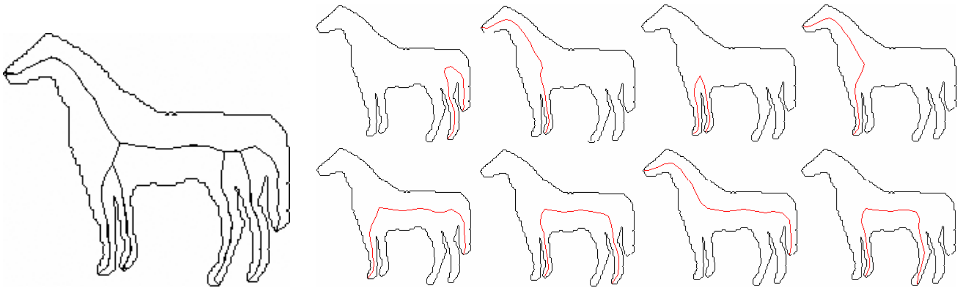
Fig. 3. (a) The original skeleton without pruning. (b) The skeleton pruned with contour partitioning.

computed with respect to the actual partially simplified versions of the polygon. Therefore, the remaining skeleton branches are determined in the context of the whole shape, e.g. the same branch that may be irrelevant for one shape, and is removed, may be relevant for a different shape, and therefore, it will remain.

In order to obtain skeletons composed of only relevant branches, provided none are missed, an appropriate stop criterion of the DCE simplification is needed. Usually we can use the same threshold as stop criterion of DCE for the shapes in the same class, because they are very similar. Moreover, our classification is very stable to our pruning skeletons, since do not just train with only one shape for each class with Bayesian rule. Therefore, even if we get a few additional skeleton branches, the classification is not influenced.

## 4. Shape Path Representation

The endpoint in the skeleton graph is called an **end node**, and the junction point in the skeleton graph is called a **junction node**. The shortest path between a pair of end nodes on a skeleton graph is called a **skeleton path**, e.g. see Fig. 4.



(a) (b)

Fig. 4. (a) The horse's skeleton. (b) The shortest paths (in red) between the pairs of endpoints.

Suppose there are $N$ end nodes in the skeleton graph $G$ to be matched, and let $v_i$ $(i = 1, 2, \ldots, N)$ denote the $i$th end node along the shape contour in the clockwise direction. Let $sp(m, n)$ denote the shape path from $v_m$ to $v_n$. We sample $sp(m, n)$ with $M$ equidistant points, which are all skeleton points. Let $R_{m,n}(t)$ denote the radius of the maximal disk at the skeleton point with index $t$ of $sp(m, n)$. Let $R_{m,n}$ denote a vector of the radii of the maximal disks centered at the $M$ sample skeleton points on $sp(m, n)$:

$$R_{m,n} = (R_{m,n}(t))_{t=1,\ldots,M} = (r_1, r_2, \ldots, r_M). \tag{1}$$

Thus, the shortest paths between every pair of skeleton endpoints are represented as sequences of radii of the maximal disks at corresponding skeleton points. In this paper, the radius $R_{m,n}(s)$ is approximated with the values of the distance transform $DT(s)$ at each skeleton point $s$. Suppose there are $N_0$ pixels in the original shape $S$. To make the proposed method invariant to the scale, we normalize $R_{m,n}(s)$ in the following way:

$$R_{m,n}(s) = \frac{DT(s)}{\frac{1}{N_0}\sum_{i=1}^{N_0} DT(s_i)}, \tag{2}$$

where $s_i$ $(i = 1, 2, \ldots, N_0)$ varies over all $N_0$ pixels in the shape.

The shape dissimilarity between two shape paths is called a **path distance**. If $R$ and $R'$ denote the vectors of radii of two shape paths $sp$ and $sp'$ respectively, the path distance $pd$ between $sp$ and $sp'$ is:

$$pd(R, R') = \sum_{i=1}^{M} \frac{(r_i - r'_i)^2}{|r_i + r'_i|}. \tag{3}$$

The main motivation for Eq. (3) is the fact that similar shapes will have similar radii sequences on their corresponding skeleton paths. Formula (3) differs from the squared Euclidean norm by the scaling factor in the denominator, which has the effect of weighting the radii difference with respect to the radii values, e.g. if both radii are large, their difference must be significant. This is motivated by human perception, since the difference in thicker parts of objects must be more significant in order to be noticed. Path distance can also be used for finding the correspondence between two similar shapes.[5]

## 5. Bayesian Classification

Compared to the method in Ref. 26, which uses contour segments and Bayesian classification to perform a recognition task, our method uses paths instead of contour segments. Since paths are normalized, our method does not require any invariant reference frame, and consequently the process of PCA[26] can be removed.

For a given query shape and a given shape class, we compute the probability that the shape belongs to the class. This step is repeated for all shape classes, and the query shape is then assigned to the class with the highest probability.

Given a shape $\omega'$ that should be classified by Bayesian classifier, we build the skeleton graph $G(\omega')$ of $\omega'$ and input $G(\omega')$ as the query. For a skeleton graph $G(\omega')$, if the number of end nodes is $n$, the corresponding number of paths is $n(n-1)/2$ compared to the number of parts $n!$ in Ref. 26. Then, the Bayesian classifier computes the posterior probability of all classes for each path $sp' \in G(\omega')$. By accumulating the posterior probability of all paths of $G(\omega')$, the system automatically yields the ranking of class hypothesis for the query shape $\omega'$.

We use Gaussian distribution to compute the probability $p$ that two skeleton paths are similar:

$$p(sp'|sp) = \frac{1}{\sqrt{2\pi\alpha}} \exp\left(-\frac{pd(sp', sp)^2}{2\alpha}\right). \tag{4}$$

For example, this probability is high for two different paths with small $pd$ value. For different datasets, $\alpha$ should be different. In our experiments, for the dataset of Aslan and Tari,[3] $\alpha = 0.15$ and $\alpha = 0.05$ for Kimia dataset.[23]

The class-conditional probability of observing $sp'$ given that $\omega'$ belongs to class $ci$ is:

$$p(sp'|c_i) = \sum_{sp \in G(c_i)} p(sp'|sp)p(sp|c_i). \tag{5}$$

We assume that all paths within a class path set are equiprobable, therefore

$$p(sp|c_i) = \frac{1}{|G(c_i)|}. \tag{6}$$

According to the probability that the query shape belongs to a given class, the posterior probability of a class given that path $sp' \in G(\omega')$ is determined by Bayes rule:

$$p(c_i|sp') = \frac{p(sp'|c_i)p(c_i)}{p(sp')}. \tag{7}$$

Similar to the above assumption, $p(ci) = 1/M$. The probability of $sp'$ is equal to

$$p(sp') = \sum_{i=1}^{M} p(sp'|c_i)p(c_i). \tag{8}$$

Through the above formulas, we can get the posterior probability of all paths of $G(\omega')$. By summing the posterior probabilities of a class over the set of paths in the query shape, we obtain the probability that it belongs to a given class. Obviously, the biggest one, $Cm$, is the class that input shape belongs to

$$C_m = \underset{i=1,\ldots,M}{\operatorname{argmax}} \sum_{sp' \in G(\omega')} p(c_i|sp'). \tag{9}$$

## 6. Experiments

In this section, we evaluate the performance of the proposed method based on the database of Aslan and Tari.[3] We selected this database due to large variations of shapes in the same classes. As shown in Fig. 5, Aslan and Tari database includes 14 classes of articulated shapes with 4 shapes in each class. We use each shape in this database as a query, and show the classification result of our system in Fig. 6. We used leave-one-out classification, i.e. the query shape was excluded from its class.

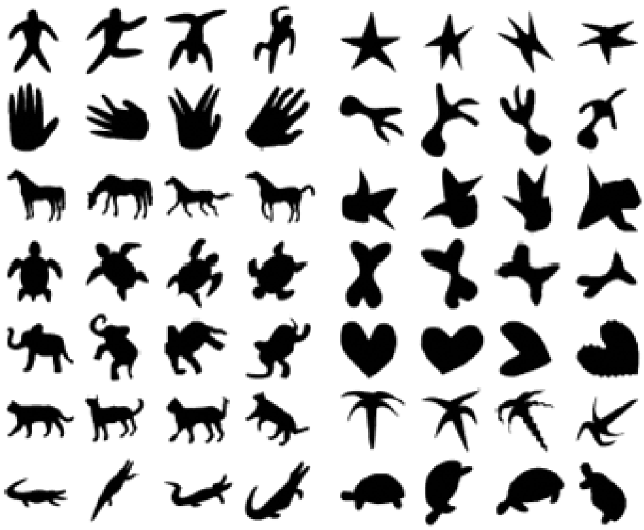


Fig. 5. Aslan and Tari database[27] with 56 shapes.

| class | query | result | query | result | query | result | query | result |
|---|---|---|---|---|---|---|---|---|
| 1 | | 1 | | 1 | | 1 | | 1 |
| 2 | | 2 | | 2 | | 2 | | 2 |
| 3 | | 3 | | 3 | | 3 | | 3 |
| 4 | | 4 | | 4 | | 4 | | 4 |
| 5 | | 5 | | 5 | | 5 | | 5 |
| 6 | | 6 | | 6 | | 6 | | 6 |
| 7 | | 7 | | 7 | | 7 | | 7 |
| 8 | | 8 | | 8 | | 8 | | 8 |
| 9 | | 9 | | 9 | | 9 | | 9 |
| 10 | | 10 | | 10 | | 10 | | 10 |
| 11 | | 11 | | 11 | | 11 | | 11 |
| 12 | | 12 | | 12 | | 12 | | 12 |
| 13 | | 13 | | **8** | | 13 | | 13 |
| 14 | | 14 | | 14 | | 14 | | 14 |

Fig. 6. Results of the proposed method on Aslan and Tari database.[27] Since each class is composed of four shapes, the class of query and the result should be the same. Red numbers mark the results where this is not the case.

The table in Fig. 6 is composed of 14 rows and 9 columns. The first column of the table represents the class of each row. For each row, there are four experimental results which belong to the same class. Each experimental result has two elements. The first one is the query shape and the second one is the classification result of our system. If the result is correct, it should be equal to the first column of the row. The red numbers mark the wrong classes assigned to query objects. Since there is only one error in 56 classification results, the classification accuracy in percentage by this measure is 98.2%. In fact, the only error is reasonable. Even a human can misclassify it. The query shape is very similar to the star, which is class 8. Therefore, in some sense, we can conclude that all of our results are correct.

We compared our method to the method presented by Sun and Super in Ref. 26, their method used the same Bayesian classifier but based on contour parts. As shown in Fig. 7, their method yields four wrong results for 56 query shapes, which yields the classification accuracy of only 92.8%. Since Aslan and Tari did not present results on their entire database, we cannot directly compare the recognition rate of our method to Ref. 3. However, we were able to compare our method to the inner distance[18] on this dataset. Inner distance[18] obtains only 94.64% through the nearest neighborhood classification, though it can solve the articulated shape classification very well.

The classification time for all 56 shapes with the proposed method takes only 5 min on the PC with 1.5 GHZ CPU and 512 M RAM. In comparison, Sun and Super's method took 13 min on the same computer.

We also apply the proposed method to Kimia dataset[23] as shown in Fig. 8, which includes 18 classes, and each class consists of 12 shapes. In each experiment, we remove the query shape from the database, therefore there are 215 shapes in

| class | query | result | query | result | query | result | query | result |
|---|---|---|---|---|---|---|---|---|
| 1 | | 1 | | 1 | | **11** | | **9** |
| 2 | | 2 | | 2 | | 2 | | 2 |
| 3 | | 3 | | 3 | | **6** | | 3 |
| 4 | | 4 | | 4 | | 4 | | 4 |
| 5 | | 5 | | 5 | | 5 | | 5 |
| 6 | | 6 | | 6 | | 6 | | 6 |
| 7 | | 7 | | 7 | | 7 | | 7 |
| 8 | | 8 | | 8 | | 8 | | 8 |
| 9 | | 9 | | 9 | | 9 | | 9 |
| 10 | | 10 | | 10 | | 10 | | 10 |
| 11 | | 11 | | 11 | | 11 | | 11 |
| 12 | | 12 | | 12 | | 12 | | 12 |
| 13 | | 13 | | **8** | | 13 | | 13 |
| 14 | | 14 | | 14 | | 14 | | 14 |

Fig. 7. Results of the Sun and Super's method on Aslan and Tari database.[27] Since each class is composed of four shapes, the class of query and the result should be the same. Red numbers mark the results where this is not the case.

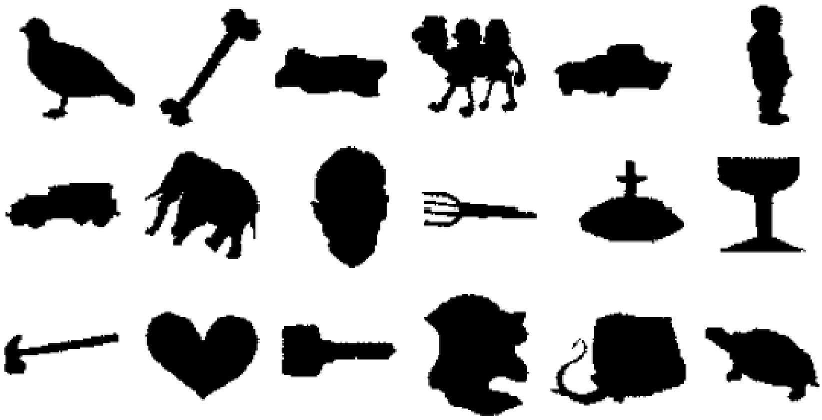


Fig. 8. Eighteen classes in Kimia dataset[23] are illustrated with one shape each.

database and one query shape. Since there are only 12 errors in 216 classification results, the classification accuracy in percentage is 94.4%. We illustrate some of our results in Fig. 9. For each class we show two example queries that are correctly classified. We also show four wrong results (chosen from the 12 error classifications) in the last two rows marked in red. Observe that some of them can be intuitively explained. For the glass, the skeleton is similar to the one of the bone. Moreover, the turtle exhibits some similarity to elephants, and the last query has similar shape to bricks.

We have also evaluated the Sun and Super's method on this dataset. Its classification accuracy is 97.2%, which means there are six wrong classification results. Four wrongly classified shapes by Sun and Super's method are shown in Fig. 10.

The classification time for all 216 shapes with the proposed method takes about 25 min on the PC with 1.5 GHZ CPU and 512 M RAM compared to Sun and Super's 45 min.

The results demonstrate that our method can perform better on the articulated shapes than Sun and Super's method, since the shapes of Aslan and Tari's dataset are more articulated than the ones in Kimia's dataset. The main reason is that the shape descriptor of the proposed method is based on the skeleton which is much more stable for articulated shapes than the contour. Though the accuracy of Sebastian *et al.*[24] on the dataset is 100% which is better than the proposed method, the proposed method is still promising and the complexity is much lower than Sebastian *et al.*

## 7. Conclusions

In this paper, we propose a novel method to classify the whole shape that is based on statistics of dissimilarities between shortest skeleton paths. The result of two different datasets demonstrated that skeleton paths are very efficient shape

| Query | Result | Query | Result |
|---|---|---|---|
| | Bone | | Bone |
| | Glass | | Glass |
| | heart | | heart |
| | Misk | | Misk |
| | Bird | | Bird |
| | Brick | | Brick |
| | camel | | camel |
| | car | | car |
| | child | | child |
| | Classic car | | Classic car |
| | elephant | | elephant |
| | face | | face |
| | fork | | fork |
| | fountain | | fountain |
| | hammer | | hammer |
| | key | | key |
| | ray | | ray |
| | turtle | | turtle |
| | camel | | bone |
| | elephant | | brick |

Fig. 9. Part of the classification results on Kimia dataset.

| Query | Results | Query | Results | Query | Results | Query | Results |
|---|---|---|---|---|---|---|---|
| | elephant | | elephant | | elephant | | camel |

Fig. 10. Part of the wrong classification results of Sun and Super's method on Kimia dataset.

representation for classification. In the future, our work will focus on combining the contour and skeleton information by combining their corresponding Bayesian classifiers.

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