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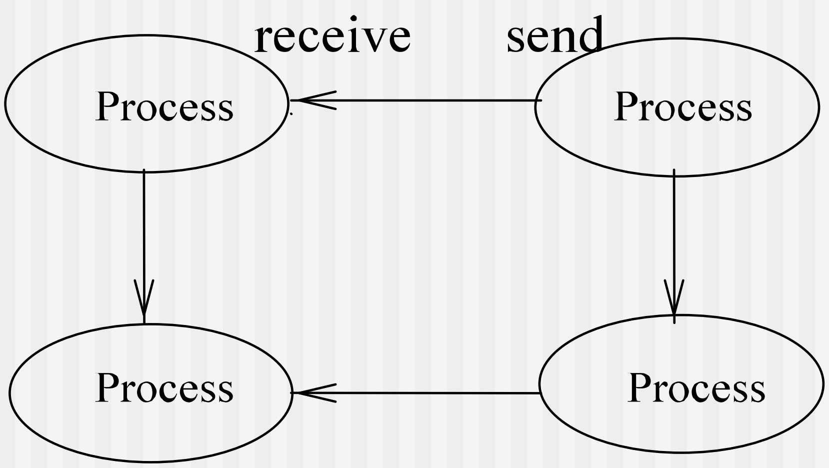
State Model

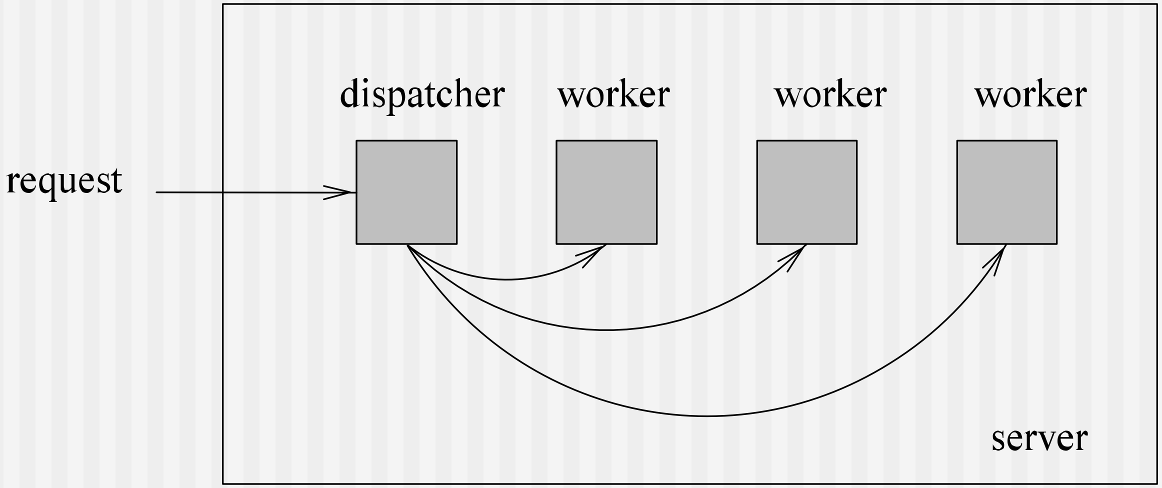
- A **process** executes three types of events: **internal** actions, **send** actions, and **receive** actions.
- A **global state (**also **configuration)**: a collection of local states and the state of all the communication channels.
- Global state evolves by means of **transitions**
- **Initiator**: first event
- **Distributed algorithm**: multiple initiators



System structure from logical point of view.

Thread

- lightweight process (maintain minimum information in its context)
- multiple threads of control per process
- multithreaded servers (vs. single-threaded process)



A multithreaded server in a dispatcher/worker model.

Preliminary

Assertions: a predicate on the configurations of an algorithm

Invariant, such as loop invariant, is an assertion

e.g., {I} **while** c body {¬c ∧ I} (under Floyd-Hoare logic)

calculate sum: 1+2+…+n, two assertions I: 1+2+…+k and c: $k < n$

Safety property: if it is true in each reachable configuration

i.e., something bad will never happen (e.g., absence of deadlock, mutual exclusion, partial correctness)

Liveness property: if executions, from some point on, contain a configuration in which the assertion holds

i.e., something good will eventually happen (e.g., fairness, termination)

Fair: if every event that can happen in infinitely many times is performed infinitely often

Complexity: time, space, message (bit) complexity

Happened-Before Relation

The **happened-before relation** (denoted by $\rightarrow$) is defined as follows:

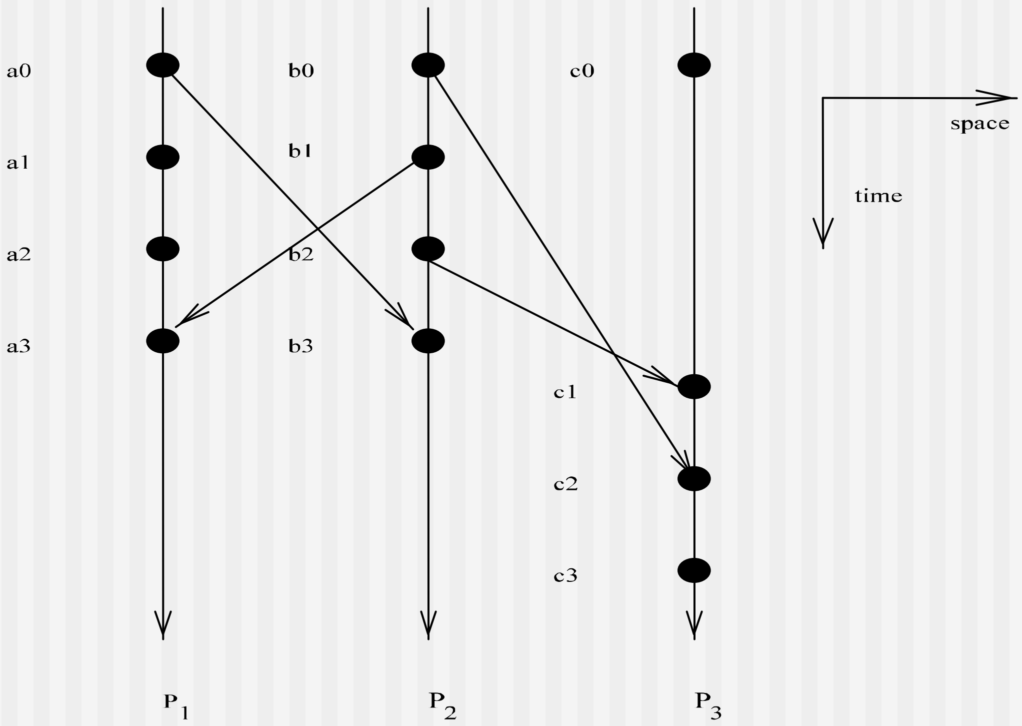
- Rule 1 : If a and b are events in the same process and a was executed before b, then $a \rightarrow b$.
- Rule 2 : If a is the event of sending a message by one process and b is the event of receiving that message by another process, then $a \rightarrow b$.
- Rule 3 : If $a \rightarrow b$ and $b \rightarrow c$, then $a \rightarrow c$.

Relationship Between Two Events

- Two events a and b are **causally related** if $a \rightarrow b$ or $b \rightarrow a$.

- Two distinct events a and b are said to be **concurrent** if a $\not\rightarrow$ b and $b \not\rightarrow a$ (denoted as $a \parallel b$).

Example 2



A time-space view of a distributed system.

Example 2 (Cont'd.)

- Rule 1:

  $a_0 \rightarrow a_1 \rightarrow a_2 \rightarrow a_3$

  $b_0 \rightarrow b_1 \rightarrow b_2 \rightarrow b_3$

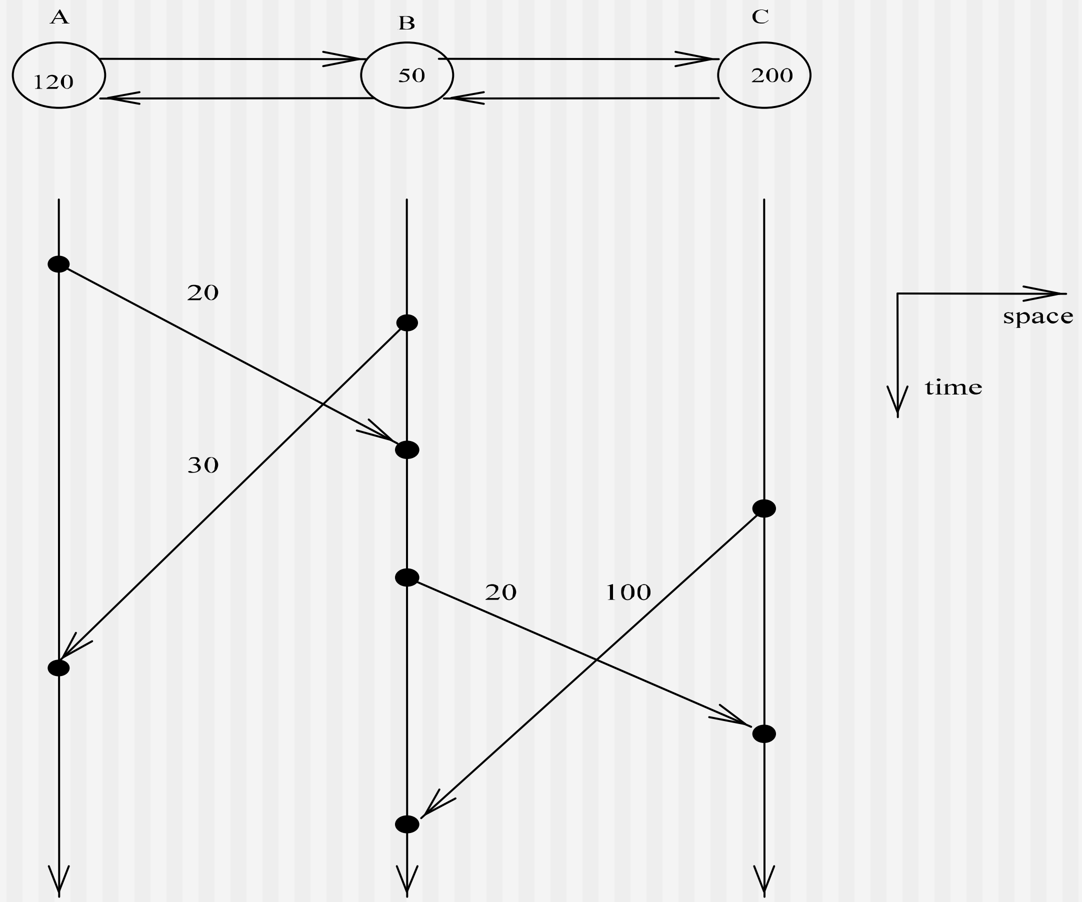
  $c_0 \rightarrow c_1 \rightarrow c_2 \rightarrow c_3$

- Rule 2:

  $a_0 \rightarrow b_3$

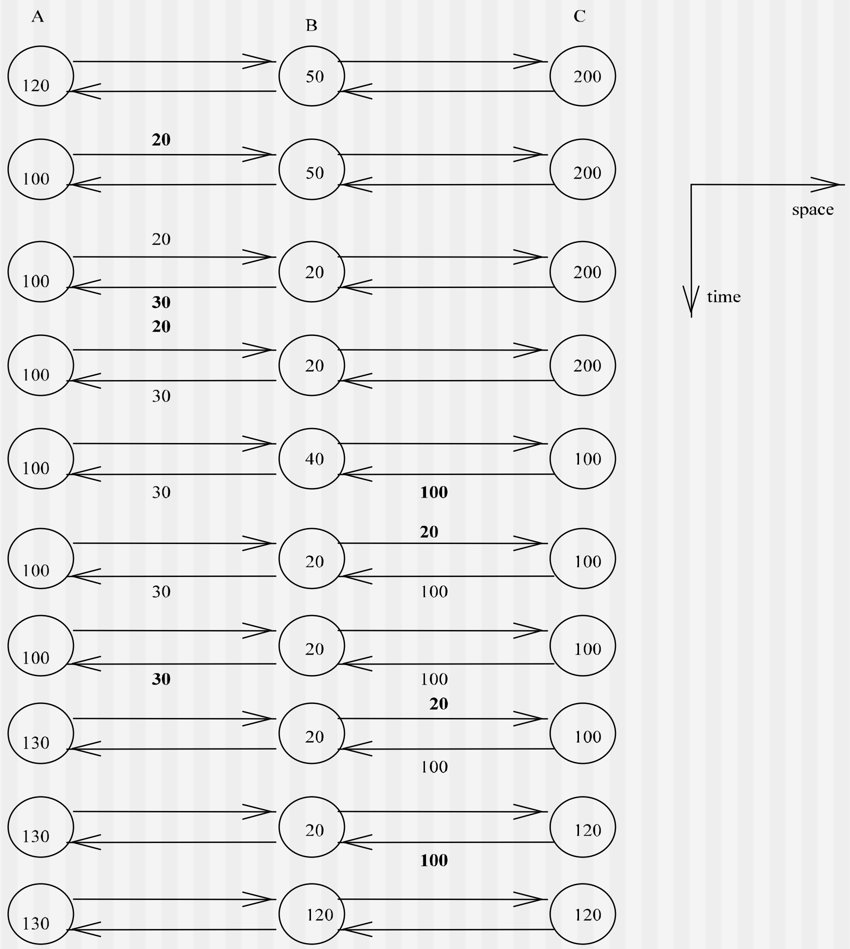
  $b_1 \rightarrow a_3$, $b_2 \rightarrow c_1$, $b_0 \rightarrow c_2$

Example 3



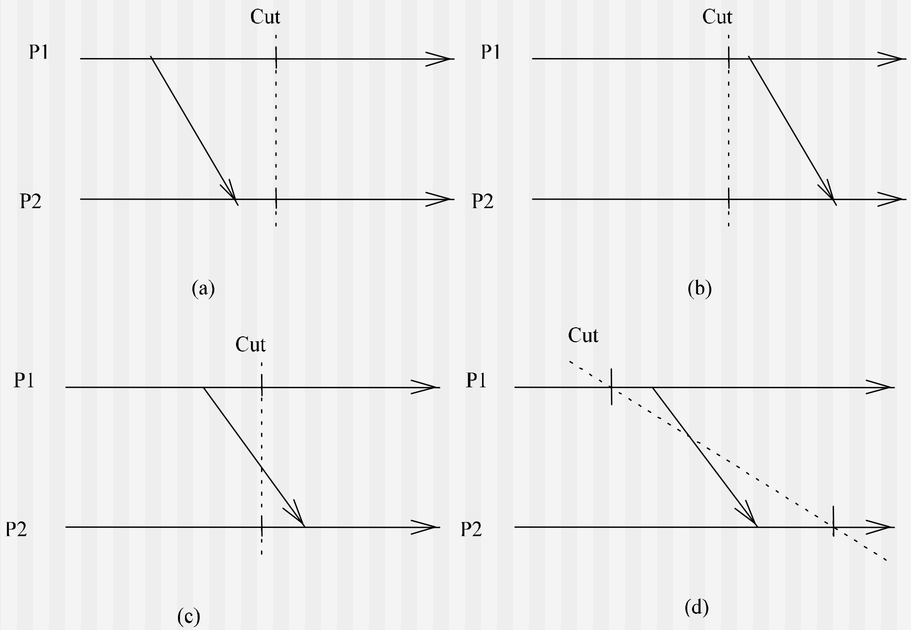
An example of a network of a bank system.

Example 3 (Cont'd.)



A sequence of global states.

Consistent Global State



Four types of cut that cross
a message transmission line.

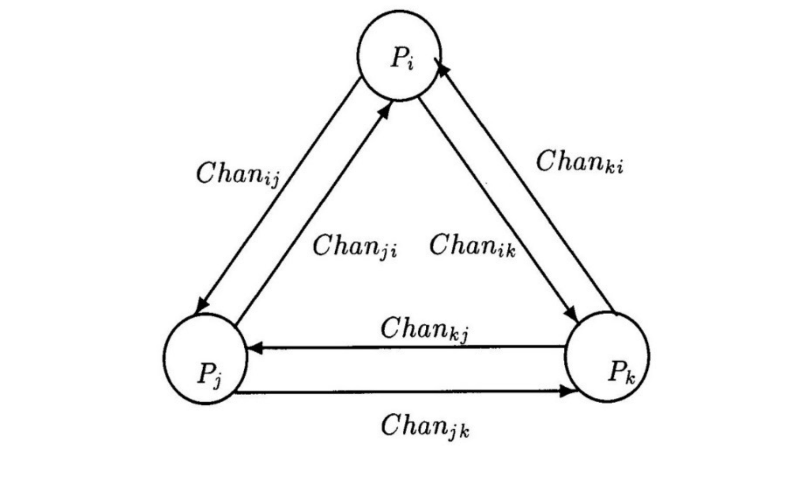
Consistent Global State (Cont'd.)

A **cut** is consistent iff no two cut events are causally related.

- **Strongly consistent**: no (c) and (d).
- **Consistent**: no (d) (orphan message).
- **Inconsistent**: with (d).

Focus 3: Snapshot of Global States

A simple distribute algorithm to capture a consistent global state.



A system with three processes P_i, P_j , and P_k.

Many key concepts: asynchronous computation, global state, information propagation and gathering, …

Chandy and Lamport's Solution

- Rule for sender *P* :

  [*P* records its local state

  ||*P* sends a marker along all the channels on which a marker has not been sent.

  ]

- Rule for receiver *Q*:

  /* on receipt of a marker along a channel *chan* */

  [*Q* has not recorded its state →

  [record the state of *chan* as an empty sequence and follow the "Rule for sender"

  ]

  □ *Q* has recorded its state →

  [record the state of *chan* as the sequence of messages received along *chan* after the latest state recording but before receiving the marker

  ]

  ]

Chandy and Lamport's Solution (Cont'd.)

- It can be applied in any system with FIFO channels (but with variable communication delays).
- The initiator for each process becomes the parent of the process, forming a spanning tree for result collection.
- It can be applied when more than one process initiates the process at the same time.

Chandy and Lamport's Solution (Cont'd.)

- Distributed algorithm: message-passing
- Distributed snapshot
- Dynamic spanning tree
- Asynchronous systems
- Message dissemination
- Progress termination
- Program debugging
  - Breakpoint
- Simulation
  - Physical and logical processes (event-driven)

Synchronous vs. Asynchronous Systems

Asynchronous Systems:

- Each node is driven by its own (independent) local clock.
- The transmission delay is finite but unpredictable.

Synchronous Systems:

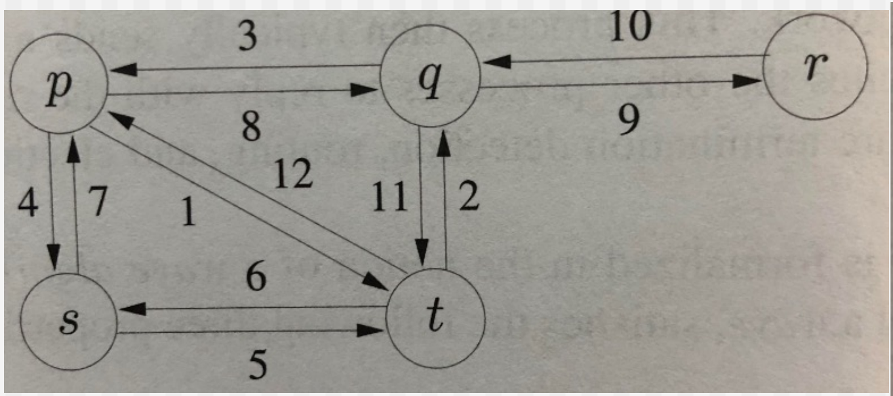
- All nodes are driven by the global clock, which generates intervals (also rounds) of fixed, nonzero duration.
- The transmission delay is nonzero, but strictly less than the duration of an interval.

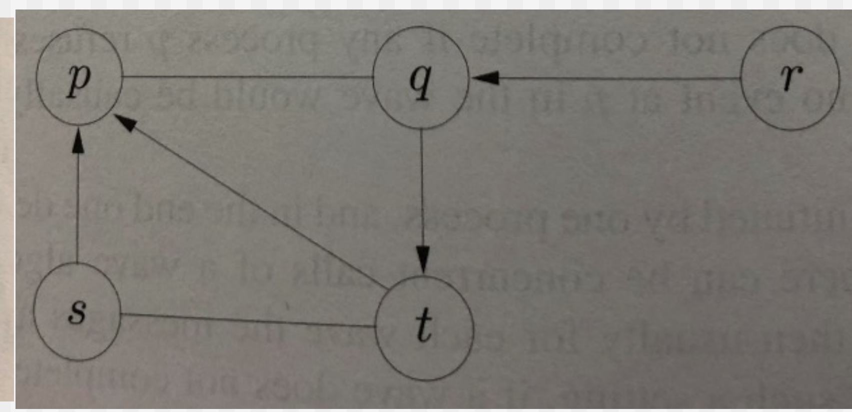
Distributed Algorithms: Traversal

Tarry's algorithm:

- A process forwards the token through the same channel once.
- A process forwards the token to its parent only when there is no other option.

Complexity: 2E messages and at most 2E time units.





Distributed Algorithms: Traversal (cont'd)

Extensions to avoid visited nodes:

- Include the IDs of visited nodes

  Complexity: 2(N-1) in time and in messages, but O(N log N) in bit complexity

- **Awerbuch**'s extension: the first-time process with the token informs its neighbors

  Complexity: 4N-2 in time and 4E in messages

- **Cidon**'s extension: improves on Awerbuch's extension

  Complexity: 2(N-1) in time and 4E in messages

Distributed Algorithms: Wave-and-Echo

Wave-and-Echo algorithm (also for counting connected nodes)

- **Initiator** starts by sending a token to all its neighbors.
- When a node receives a token for the first time, it makes the sender its parent, and sends the token to all its neighbors.
- When a node has received messages from all its neighbors, it sends a message to its parent.
- When the **initiator** has received messages from all its neighbors, it stops.

General **wave (-and-echo)** algorithm (also for information propagation)

- A process often needs to gather information from all other processes.
- Usually the process starts with an initiator and ends with the same imitator (after collecting all data/results from all other processes).
- When the wave algorithm is issued at multiple nodes. Many waves, except one, will fail

Distributed Algorithms: Termination

Dijkstra-Scholten (tree-based):

- The initiator of the root of the tree.
- Upon receiving a message:
  - If the receiving process is currently not in the tree: the process joins the tree by becoming a child of the sender.
  - If the receiving process is already in the computation: the process immediately sends an acknowledgment message to the sender.
- When a process has no more children and has become idle, the process detaches itself from the tree by sending an acknowledgment to its tree parent.
- Termination occurs when the initiator has no children and has become idle.

Example: global snapshot (with one king)

Distributed Algorithms: Termination (cont'd)

Shavit-Francez (forest-based):

- Same as Dijkstra-Scholten, except with multiple initiators.
- Each non-initiator joining one tree.
- Termination detection initiated by multiple initiators through a wave algorithm

Example: global snapshot (with multiple kings)

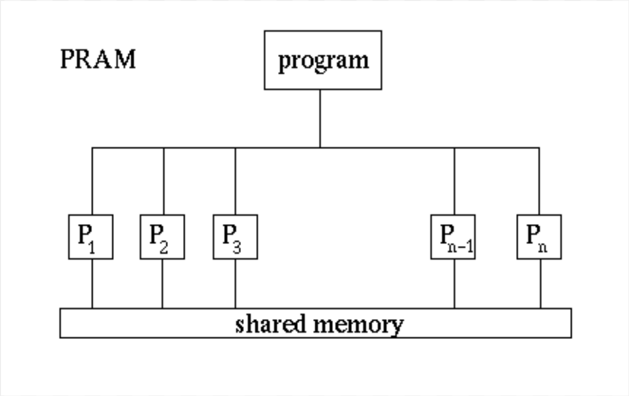
Other termination algorithms:

- **Weight-throwing** algorithm: dividing a fixed weight over the active processes
- **Rana**'s algorithm: waves tagged with logical clocks
- **Safra**'s algorithm: token-based traversal

Other Algorithms: Parallel Algorithms

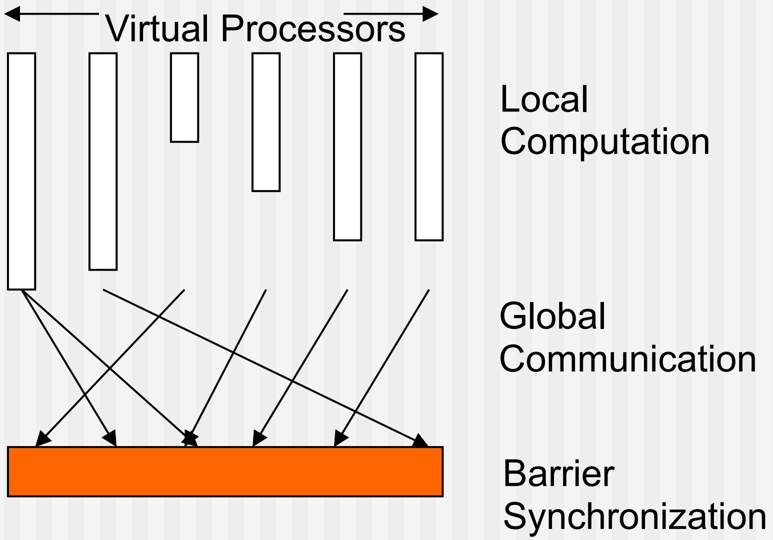
PRAM model

- Parallel random access memory
- EREW, ERCW, CREW, CRCW models
- Chap. 2 of JaJa's
  "an introduction to parallel algorithms"



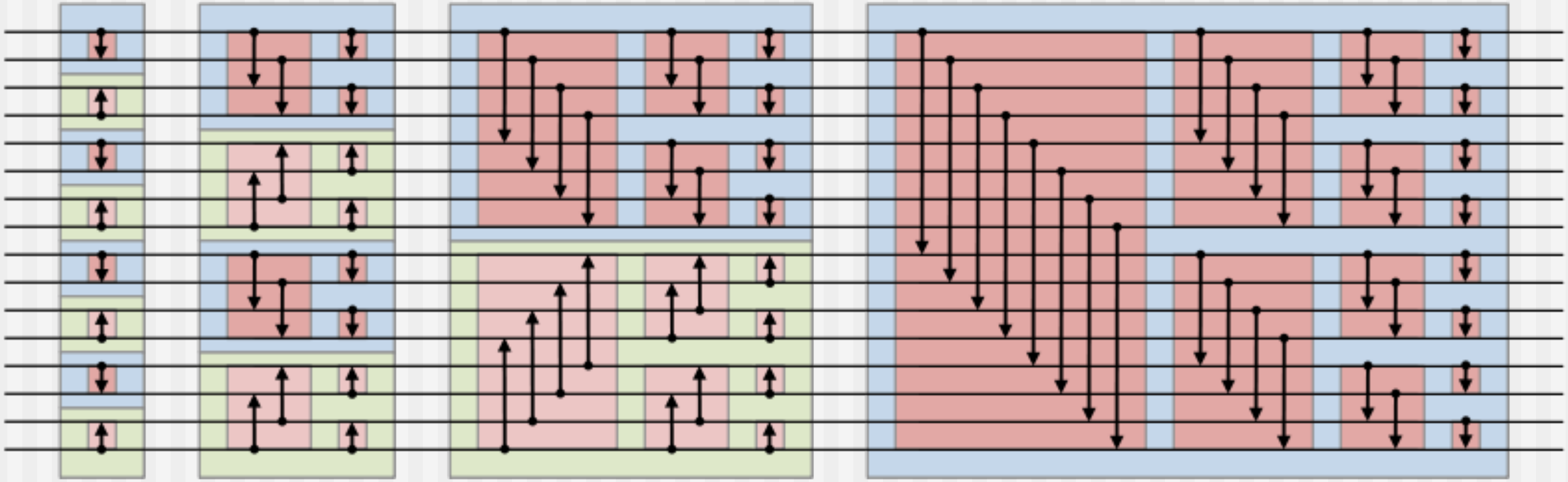
BSP model by L. Valiant (1990)

- Bulk synchronous parallel (BSP)
- Sequential composition of "supersteps"
  - Local computation
  - Process communication
  - Barrier synchronization



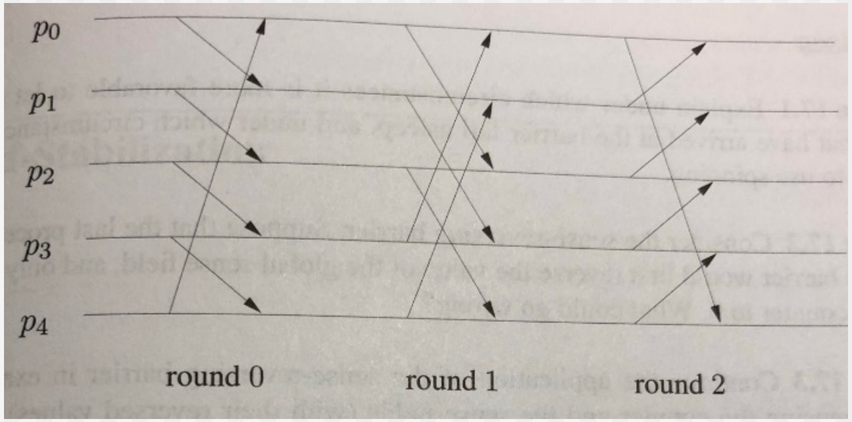
Parallel Algorithm: Bitonic sorter by K. Batcher

- Sorting network based on **Bitonic sequence**
  - Up-then-Down or Down-then-Up
  - $O(n \log^2(n))$ comparators
  - $O(\log^2(n))$ latency
- Also Batcher's **odd-even sort** (small -> large)



Barrier Synchronization

- **Sequential**: One process p (leader, through leader election if needed)
  - Process p issues wave-and-echo to all nodes
  - Process p indicates next round to all nodes
- **Parallel**: Processes p_0, p_1, …, p_{N-1}, n starts from 0 until $\log_2 N - 1$
  - Notifies process $p_{(i+2^n)}$ mod N,
  - Waits for notification by process $P_{(i-2^n)}$ mod N, and
  - Processes to round n+1



Focus 4: Lamport's Logical Clocks

Based on a "**happen-before**" relation that defines a **partial order** on events

- *Rule*$_1$. Before producing an event (an external send or internal event), we update *LC* :

  $$LC_i = LC_i + d \qquad (d > 0)$$

  (d can have a different value at each application of *Rule*$_1$)
- *Rule*$_2$. When it receives the time-stamped message (m, LC_j , j), P_i executes the update

  $$LC_i = \max\{Lc_i, LC_j\} + d \quad (d > 0)$$

Focus 4 (Cont'd.)

A **total order** based on the partial order derived from the happen-before relation

$$a \text{ (in } P_i \text{)} \Rightarrow b \text{ (in } P_j \text{)}$$

iff

(1) $LC(a) < LC(b)$ or (2) $LC(a) = LC(b)$ and $P_i < P_j$ where < is an arbitrary total ordering of the process set, e.g., <can be defined as $P_i < P_j$ iff $i < j$.

A total order of events in the table for Example 2:

$$a_0\ b_0\ c_0\ a_1\ b_1\ a_2\ b_2\ \mathrm{a}_3\ b_3\ c_1\ \mathrm{c}_2\ c_3$$

Vector and Matrix Logical Clock

Linear clock: if $a \rightarrow b$ then $LC_a < LC_b$

Vector clock: $a \rightarrow$ b iff $LC_a < LC_b$

Each P_i is associated with a vector $LC_i[1..n]$, where

- $LC_i[i]$ describes the progress of P_i, i.e., its own process.
- $LC_i\ [j]$ represents P_i's knowledge of P_j's progress.
- The $LC_i[1..n]$ constitutes P_i's local view of the logical global time.

Vector and Matrix Logical Clock (Cont'd.)

When $d = 1$ and $init = 0$

- $LC_i[i]$ counts the number of internal events
- $LC_i[j]$ corresponds to the number of events produced by P_j that causally precede the current event at P_i.

Knowledge and implicitly knowledge

Vector and Matrix Logical Clock (Cont'd.)

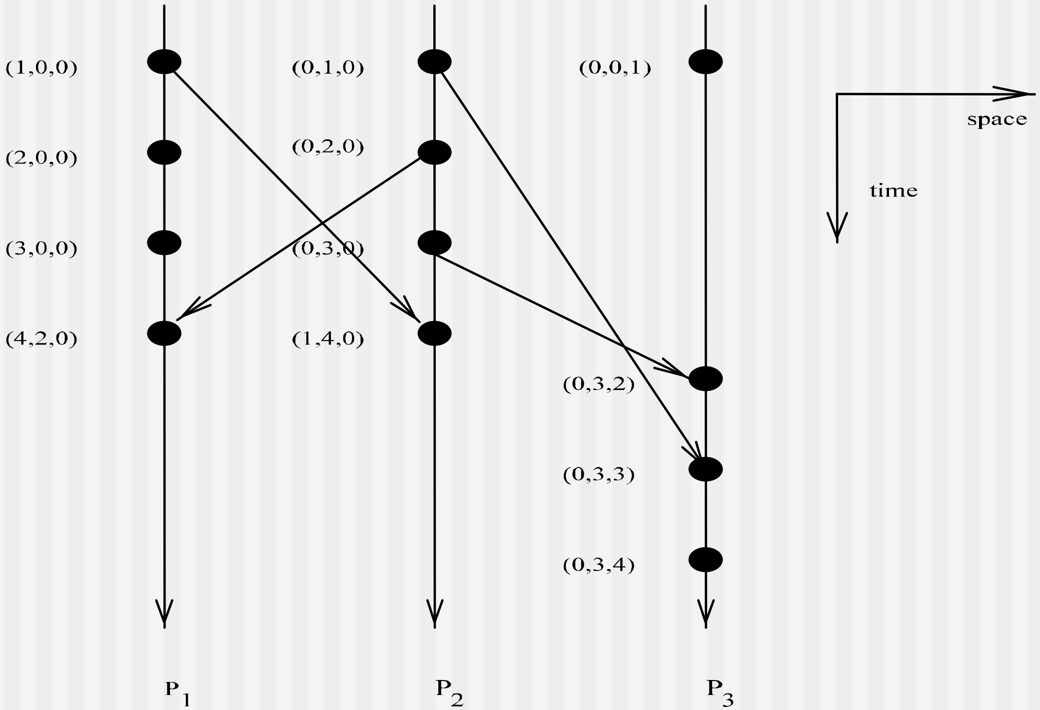
- *Rule*$_1$. Before producing an event (an external send or internal event), we update $LC_i[i]$:

$$LC_i[i] := LC_i[i] + \mathrm{d} \quad (d > 0)$$

- *Rule*$_2$. Each message piggybacks the vector clock of the sender at sending time. When receiving a message (m, LC_j , j), P_i executes the update.

$$LC_i[k] := \max\,(LC_i[k]; LC_j[k]),\ 1 \leq \mathrm{k} \leq n$$
$$LC_i[i] := LC_i[i] + d$$

Example 4



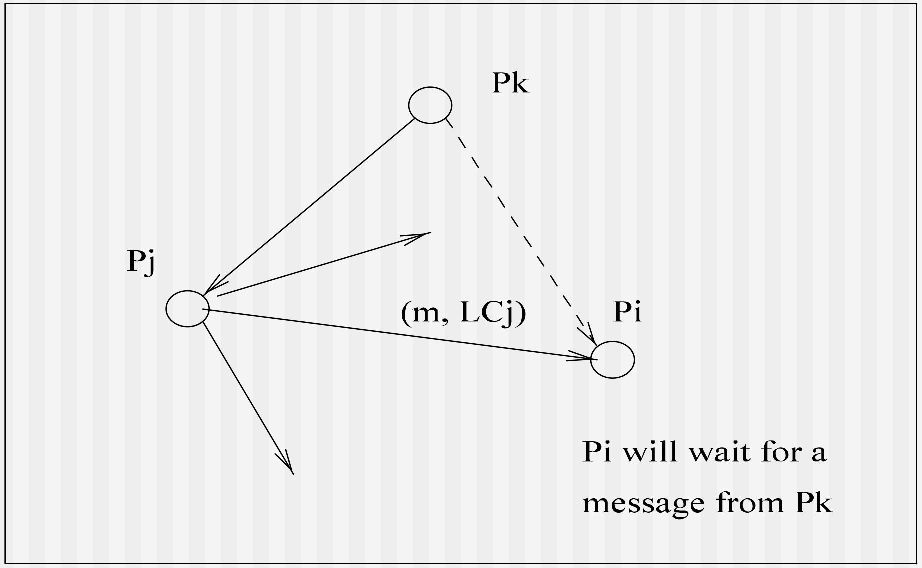
An example of vector clocks.

Example 5: Totally-Ordered Multicasting

- Two copies of the account at A and B (with balance of $10,000).
- Update 1: add $1,000 at A.
- Update 2: add interests (based on 1% interest rate) at B.
- Update 1 followed by Update 2: $11,110.
- Update 2 followed by Update 1: $11,100.

Example 6: Application of Vector Clock

Internet electronic bulletin board service



Network News.

When receiving *m* with vector clock LC_j from process *j*, P_i inspects timestamp LC_j and will postpone delivery until all messages that causally precede *m* have been received.

Matrix Logical Clock

Each P_i is associated with a matrix $LC_i[1..n, 1..n]$ where

- $LC_i[i, i]$ is the local logical clock.
- $LC_i[k, l]$ represents the view (or knowledge) P_i has about P_k's knowledge about the local logical clock of P_l.

If

$$\min(LC_i[k, i]) \geq t$$

then P_i knows that every other process knows its progress until its local time t.

Physical Clock

- Correct rate condition:

  $\forall_i \ |dPC_i(t)/\ dt - 1\ | < \alpha$

- Clock synchronization condition:

  $\forall_i \ \forall_j \ |PC_i(t) - PC_j(t)| < \beta$

Lamport's Logical Clock Rules for Physical Clock

- For each i, if P_i does not receive a message at physical time t, then PC_i is differentiable at t and $dPC(t)/dt > 0$.
- If P_i sends a message m at physical time t, then m contains $PC_i(t)$.
- Upon receiving a message (m, PC_j) at time t, process P_i sets PC_i to maximum $(PC_i(t - 0), PC_j + \mu_m)$ where μ_m is a predetermined minimum delay to send message m from one process to another process.

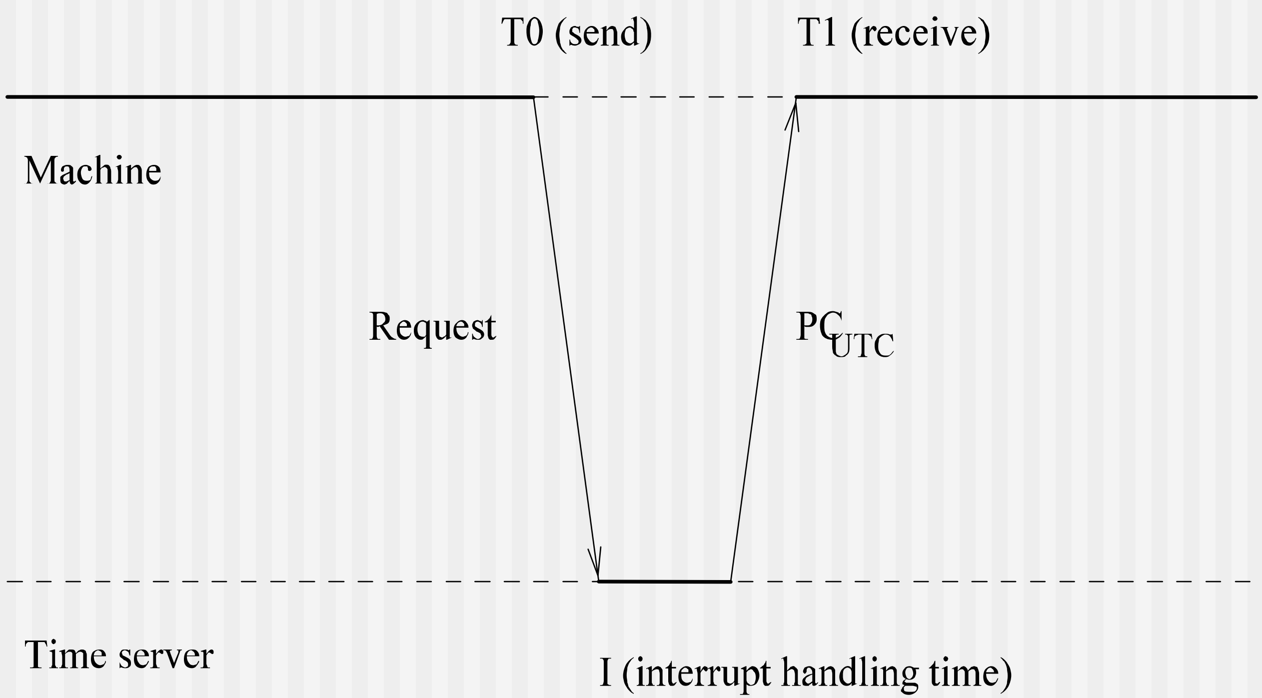
Focus 5: Clock Synchronization

- UNIX **make** program:
  - Re-compile when *file.c*'s time is large than *file.o*'s.
  - Problem occurs when source and object files are generated at different machines with no global agreement on time.
- **Maximum drift rate** ρ : $1-\rho \leq dPC/dt \leq 1+\rho$
  - Two clocks (with opposite drift rate ρ) may be $2\rho\Delta t$ apart at a time Δ after last synchronization.
  - Clocks must be resynchronized at least every $\delta/2\rho$ seconds in order to guarantee that they will be differ by no more than δ.

Cristian's Algorithm

- Each machine sends a request every $\delta/2\rho$ seconds.
- Time server returns its current time PC_{UTC} (UTC: Universal Coordinate Time).
- Each machines changes its clock (normally set forward or slow down its rate).
- Delay estimation: $(T_r - T_s - I)/2$, where T_r is receive time, T_s send time, and I interrupt handling time.

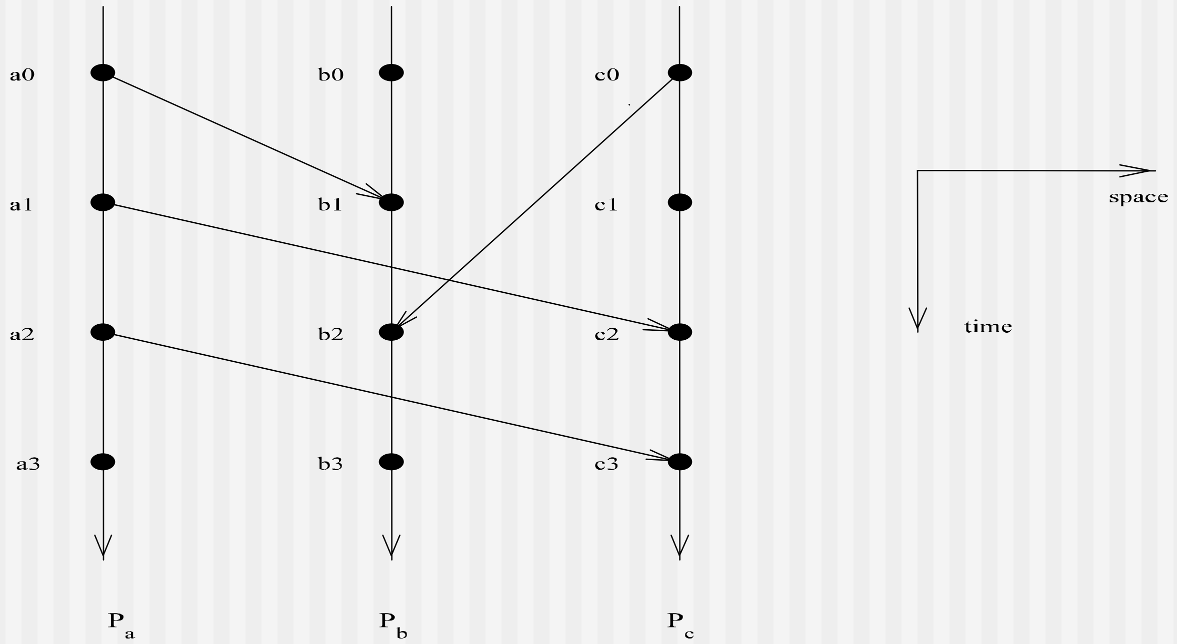
Cristian's Algorithm (Cont'd.)



Getting correct time from a time server.

Exercise 2

1. Consider a system where processes can be dynamically created or terminated. A process can generate a new process. For example, P_1 generates both P_2 and P_3. Modify the happened-before relation and the linear logical clock scheme for events in such a dynamic set of processes.
2. For the distributed system shown in the figure below.



Exercise 2 (Cont'd)

- Provide all the pairs of events that are related.
- Provide logical time for all the events using
  - linear time, and
  - vector time
  - Assume that each LC_i is initialized to zero and $d = 1$.

3. Provide linear logical clocks for all the events in the system given in Problem 2. Assume that all *LC*'s are initialized to zero and the *d*'s for P_a, P_b, and P_c are 1, 2, 3, respectively. Does condition $a \rightarrow b \Rightarrow LC(a) < LC(b)$ still hold? For any other set of *d*'s? and why?
4. Traversal on graph {(a, b), (b, c), (b, d), (c, e), (d, e), (e, f)} using Terry's solution and Awerbuch's extension.
5. Show details of sorting (4, 6, 1, 3, 5, 8, 7, 2) and (1, 4, 8, 7, 2, 6, 5, 3) on an 8-input-and-8-output Batcher's Even-Odd sorting network.