Homework 4, Due day: March 30

All solutions should be typed, using Latex preferably (suggested to use Overleaf software).

(1) Suppose we have a set of three sellers, labelled as a, b, and c, and a set of three buyers, labelled as x, y, and z. Each seller is offering a distinct house for sale, and the valuations of the buyers are as follows: Buyer x: 3 (for a), 5 (for b), 4 (for c); buyer y: 1 (for a), 7 (for b), 1 (for c); buyer z: 2 (for a), 2 (for b), 3 (for c). Describe what happens, if we run the bipartitegraph auction procedure, by saying what the prices are at the end of each round of the auction, including what the final market clearing prices are when the auction completes. (Note that there may be multiple constricted set of buyers. You can choose anyone. Will the choice matter on the final market-clearing prices?)

(2) Solve the airline scheduling problem shown in Figure 7.17 with one more added flight: PIT 9 to LAS 4. (a) Convert it first to the circulation problem. (b) Then convert the problem to the maximum flow problem. (c) Solve the problem for k=2 and k=3.

(3) In the Survey Design Problem given in in the class notes, suppose c_i, c_i' for (s, 1), (s, 2), (s, 3), (s, 4), (s, 5) are [2, 3], [1, 2], [2, 4], [1, 2], [2, 2], respectively and p_j, p_j' for (1', t), (2', t), (3', t), (4', t), (5', t) are [2, 2], [3, 3], [1, 2], [1, 1], [2, 3], respectively. Check if there is a feasible way to design the survey. If there is one, show how should the survey be designed.

(4) In the Project Selection Problem, the profits for projects a, b, c, d, e, f, and g are 8, 5, 3, -2, -1, -4 and 2, respectively. In addition, a's prerequisites are d and e, b's prerequisites are e and g, and c's prerequisite is f. Construct the prerequisite graph G and then extended prerequisite graph G'. Show how is a minimum cut found in G'. Show the corresponding feasible set of projects with maximum profit.

(5) Chapter 7, 7

(6) Chapter 7, 14