A Fault-Tolerant and Deadlock-Free Routing Protocol in 2-D Meshes Based on Odd-Even Turn Model [*]

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Abstract

We propose a deterministic fault-tolerant and deadlock-free routing protocol in 2-dimensional (2-D) meshes based on dimension-order routing and the odd-even turn model. The proposed protocol, called extended X-Y routing, *does not use any virtual channels by prohibiting certain locations of faults and destinations. Faults are contained in a set of disjointed rectangular regions called faulty blocks. The number of faults to be tolerated is unbounded as long as nodes outside faulty blocks are connected in the 2-D mesh network. The extended X-Y routing can also be used under a special convex fault region called an* orthogonal faulty block, *which can be derived from a given faulty block by activating some nonfaulty nodes in the block. Extensions to partially adaptive routing, traffic- and adaptivity-balanced using virtual networks, and routing without constraints using virtual channels and virtual networks are also discussed.*

Key words: *Deadlock-free routing, deterministic routing, fault models, fault tolerance, turn models, virtual channels.*

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1 Introduction

The direct network is a popular means to construct multicomputers, where a set of channels are used to connect each processor (node) to limited neighbors. In a multicomputer system, routing algorithms provide mechanisms for communication between nodes. The performance of such a system depends heavily on the efficiency of routing algorithms. Routing algorithms are either deterministic or adaptive. Deterministic routing uses only one path to route packets from a source to a destination, while adaptive routing makes use of many different routes. Most commercial systems use deterministic routing because of its deadlock freedom and ease of implementation.

Dimension-order routing is a commonly used deterministic routing algorithm in mesh-connected multicomputers which include meshes, tori (meshes with wraparound connections), and hypercubes. Among commercial multicomputers and research prototypes, IBM Blue Gene [11] uses a $32 \times 32 \times 32$ 3-D mesh. Alpha 21364's multiple processor network [19] employs a 2-D torus, and SGI Origin [1] adopts a variation of the hypercube structure. In dimension-order routing, a routing packet is routed in one dimension at a time (the offset between the source and destination nodes is reduced to zero along that dimension). X-Y routing is an example of dimension-order routing used in 2-dimensional (2-D) meshes and tori. In X-Y routing, the packet is routed first in the x dimension and then in the y dimension. Unfortunately, X-Y routing is not fault-tolerant and cannot tolerate even a single fault.

Designing a routing protocol that is both fault-tolerant and deadlock-free poses a major challenge. The wormhole switching technique used in the latest generation of multicomputers is subject to deadlock more than packet switching. In addition, wormhole switching tends to support routing with less fault tolerance. Wormhole routing divides a message into packets and packets into flits. It then routes flits through the network in a pipeline fashion. When the header flit reaches a node that has no output channel available, all of the flits are blocked where they are (in place). A deadlock occurs when some packets from different messages cannot advance toward their destinations because the channels requested by them are not available. All the packets involved in a deadlocked configuration are blocked forever. Figure 1 shows a deadlock situation in a 2-D mesh. Boxes represent flit buffers and the number inside each buffer indicates the destination node id. Solid arrows represent the channel requested by the head flit in the buffer. Cycles represent nodes

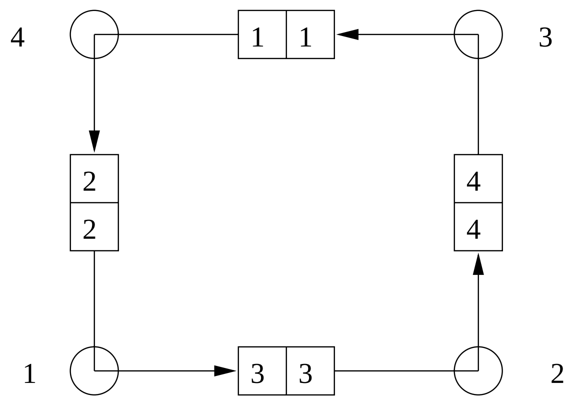

Figure 1. A deadlock situation involving four packets in a 2×2 submesh.

(and their switches). Clearly, the packets intended for 1 are blocked by the ones intended for 2. The ones intended for 2, in turn, are blocked by the ones for 3, and the ones for 3 by the ones for 4. Finally, the packets intended for 4 are blocked by the ones for 1 to complete a cyclic waiting queue – a deadlock situation occurs, where competing resources are channels (instead of nodes and switches).

Deadlock avoidance is a commonly used approach in which channels are granted to a packet in such a way that a request never leads to a deadlock. To achieve fault tolerance, faults are normally contained in a set of disjointed rectangular regions called *faulty blocks*. Each faulty block may include some nonfaulty nodes. The convexity of faulty blocks facilitates a simple design of deadlock-free routing. It is assumed that fault distribution is not known and only the boundary nodes of each faulty block know their existence. To design a deadlock-free routing, virtual channels [8] and virtual networks [18] are commonly used to provide a certain degree of routing freedom to route around a faulty block.

Ideally, a fault-tolerant and deadlock-free routing protocol should meet the following objectives: (1) Distributed routing: the routing is distributed without the knowledge of the number and distribution of faults. (2) Feasibility: the packet sent from the source should eventually reach the destination. (3) Fault tolerance: the packet should be able to bypass faults encountered during the routing process. (4) Freedom of deadlock: under no situation can several packets be involved in the deadlock situation. Ideally, the freedom of deadlock should be achieved using a minimum number of virtual channels. (5) Short route: the packet should still be routed along a short route if possible. (6) Reasonable fault model: the fault model should be realistic and should not be overly conservative (without disabling many nonfaulty nodes). (7) Adaptivity: the routing should be partially adaptive when possible. (8) Balanced traffic: channels should be used in a balanced way to avoid possible congestion in certain areas. Some of the objectives may conflict with each other; therefore, tradeoffs are needed.

The proposed fault-tolerant and deadlock-free routing protocol in 2-D meshes is based on X-Y routing and the *odd-even turn model* [7], an extension to Glass and Ni's turn model [14] where certain turns are prohibited to avoid deadlock. The protocol, called *extended X-Y routing*, is proposed to meet objectives (1) to (5). The extended X-Y routing is deterministic, and it does not use any virtual channels by prohibiting certain locations of faults and destinations. The main purpose of posing such restrictions is to better present our idea without going into messy details of boundary situations. The protocol is first proposed as a deterministic version followed by an extension to the partially adaptive one. The number of faults to be tolerated is unbounded as long as nodes outside faulty blocks are connected in the resultant mesh network. Each faulty block is surrounded by a *boundary ring* consisting of four boundary lines, one for each direction. However, the boundary line defined at the east (and west) side of each faulty block consists of two lines: one in an even column (column with an even label) and one in an odd column (see Figure 5). The faulty block is so defined that nodes on the boundary lines of a faulty block (simply called boundary nodes) do not intersect with any other faulty block, providing just enough flexibility for the packet to make appropriate turns. In the absence of faults, the extended X-Y routing works like a regular X-Y routing which routes packets along the x dimension first followed by the y dimension. When packets reach a boundary node of a faulty block, the boundary ring is used to route packets around the block.

To meet the objective (6), the extended X-Y routing is extended to be applied to a special convex fault region called an *orthogonal faulty block* [26], which can be derived from a given faulty block after activating some nonfaulty nodes in the block. The *localized algorithm* [10, 27], a special type of decentralized algorithm, is used to construct faulty blocks, orthogonal faulty blocks, and boundary lines. To meet objectives (7) and (8), extensions to partial adaptive routing, traffic- and adaptivity-balanced routing using virtual networks, and routing without constraints using virtual channels and virtual networks are also discussed.

The following assumptions are used in this paper: (1) Only node faults are considered, and they are contained in a set of disjointed faulty blocks defined in the paper. (2) The dynamic fault model is used; however, it is assumed that no new faults occur during a routing process. (3) Both the source and destination nodes are outside any faulty block. In addition, the destination is not a boundary node of any faulty block. (4) Faults do not appear at four edges of a mesh. In addition, no fault appears in two columns that are adjacent to the west and east edges of the mesh. (5) There is no limit on the number of faults as long as the 2-D mesh is connected.

Note that almost all the above assumptions can be relaxed. Related to condition (1), link faults can be treated as node faults by considering their end nodes faulty, although the "inflation" of faults might occur. Since faulty blocks converge quickly when new faults occur (or when faulty nodes recover), the proposed approach can be extended to handle dynamic faults during a routing process through a careful design. Condition (3) can be removed by using two virtual channels as will be shown in the extension of the proposed approach. Condition (4) is used to avoid handling the complex boundary situation. If condition (4) fails, either nodes of the corresponding edge(s) are disabled and removed from the mesh or virtual channels are introduced as used in many existing approaches [3, 23] to route around faulty blocks that are at the edge of the mesh.

The remainder of the paper is organized as follows: Section 2 discusses related work. Section 3 provides preliminaries where the odd-even turn model is reviewed, the general methodology of localized algorithms is discussed, and an extended faulty block model is introduced. Section 4 proposes the extended X-Y routing, which is a fault-tolerant and deadlock-free routing protocol and uses no virtual channels. Section 5 extends the protocol to a 2-D mesh with orthogonal faulty blocks. A localized algorithm for the formation of orthogonal faulty blocks is also included. Section 6 lists ideas for other possible extensions. Section 7 concludes the paper and discusses possible future work.

2 Related Work

Virtual channels [8] were first introduced to prevent deadlock and offer adaptivity in routing, rather than for fault tolerance. Normally, several virtual channels are multiplexed across a physical channel. Duato [9] provided a general deadlock-free routing approach by introducing the notion of escape channels. In this general approach, virtual channels are divided into two groups: one for nonminimal adaptive routing and the other (called escape channels) for minimal, deterministic routing. Park and Agrawal [20] discussed a similar design methodology for deadlock-free routing, but routing functions are based on the history of channels in addition to destination information. Fleury and Fraigniand [12] gave a comprehensive survey on different deadlock-free routing protocols.

Linder and Harden [18] were the first to use virtual channels and virtual networks to achieve fault tolerance. Their method requires $O(2^n)$ virtual channels for a fully adaptive fault-tolerant routing in an n-D mesh. Using virtual channels has some disadvantages, for example, routers based on virtual channels require more gates and time compared with those not based on virtual channels. To reduce the number of virtual channels, Chien and Kim [6] introduced the planar adaptive routing which provides partial adaptivity in an n-D mesh by first dividing the routing process into a sequence of phases and then forwarding packets in two dimensions within each phase.

Many fault-tolerant routing algorithms in 2-D meshes and tori have been proposed. In Boppana and Chalasani's approach [3], fault regions are surrounded by either fault rings or fault chains (used when the faulty block is at the edge of a mesh). When a packet encounters a fault region, a fault ring or chain is used to route the packet around the region. Deadlock is avoided by using four virtual channels per physical channel for dimension-order routing. Many extensions based on Boppana and Chalasani's approach have been proposed [4, 5, 16, 21, 23, 24, 28]. These extensions try to reduce either the number of nonfaulty nodes in a faulty block by considering different types of fault regions or the number of virtual channels. So far the best results can reduce the number of virtual channels to two or three depending on the type and distance between faulty blocks used. To our knowledge, there is no deadlock-free dimension-order routing that can tolerate an unlimited number of faults without using virtual channels.

Glass and Ni's fault-tolerant routing [13] in meshes without using virtual channels is based on the turn model [14]. However, its fault tolerance capability is limited to $n - 1$ in an n-D mesh, i.e., one fault in a 2-D mesh. Fault-tolerant routing without using virtual channels exists for non-dimension-order routing. For example, fault-tolerant path-based routing [17] is based on finding a Hamiltonian path or pseudo Hamiltonian path in a faulty mesh or torus. However, it is nonminimal, and unlike the faulty block model, path information is difficult to maintain in a localized way. Note that routing that allows backtracking can potentially tolerate an unlimited number of faults such as the one proposed by Suh et al [22]. In such an approach, routing history is coded in the header to navigate the routing process.

Recently, Ho and Stockmeyer [15] proposed a new approach to fault-tolerant routing for mesh-connected multicomputers with an objective of minimizing the number of "turns" in each route. Instead of using the rectangular faulty block model, some nonfaulty nodes are disabled so that all the remaining nodes can be reached through 2 rounds of dimension-order routing. Disabled nodes are called *sacrificial lambs* and are similar to nonfaulty nodes included in faulty blocks. In this approach, two virtual channels are used. A 2-approximation greedy algorithm is proposed to find a small set of disabled nodes.

Although routers with many virtual channels are quite common today [19], the gain of reducing the overhead of additional virtual channels still exists. For example, the arbitration mechanism is simplified with a reduced number of channels. One can envision an architecture for multicomputers similar to the Infiniband architecture [2] in which separated virtual channels are used for each service level (different class). In such a case, more service levels can be supported if a fault-tolerant and deadlock-free routing algorithm is supported without additional virtual channels.

3 Preliminaries

In this section, we first review the mesh topology and Chiu's odd-even turn model which is an extension to Glass and Ni's turn model. We then discuss the general methodology of localized algorithms. Finally, we introduce an extended faulty block model.

3.1 2-D meshes

A 2-dimensional (2-D) mesh with n^2 nodes has an interior node degree of 4. Each node u has an address u: (u_x, u_y), where $u_x, u_y \in \{0, 1, 2, ..., n-1\}$. Two nodes u: (u_x, u_y) and v: (v_x, v_y) are connected if their addresses differ in one and only one dimension, say x, moreover, $|u_x - v_x| = 1$. Each node, except the one at the edge of a 2-D mesh, has four neighbors, one in each of four directions: East, South, West, and North.

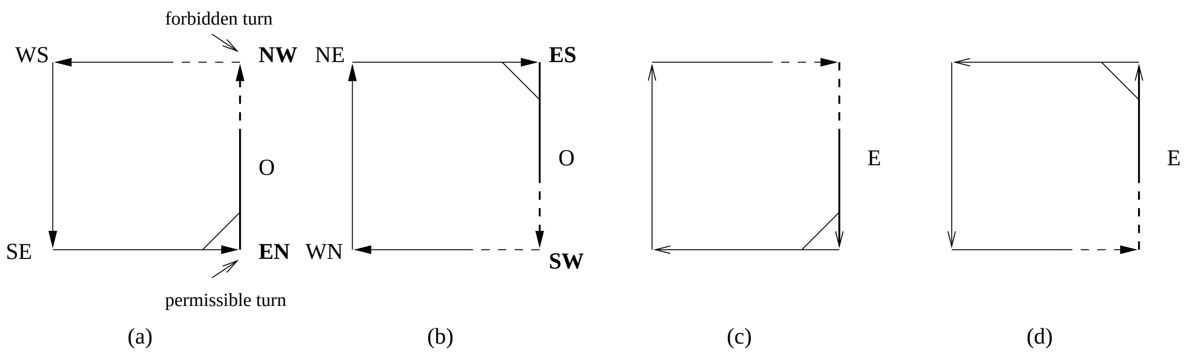

Figure 2. Permissible EN, NW, ES, and SW turns.

3.2 Odd-even turn model

Chiu [7] proposed an odd-even turn model, an extension to Glass and Ni's turn model [14]. In general, deadlock avoidance tries to avoid the formation of a cycle, which is a necessary condition for deadlock. A cycle in a mesh consists of several turns. For example, SW (south-west), WN, NE, and ES turns are essential in a clockwise cycle. The X-Y routing is made deadlock-free by prohibiting a turn from the y dimension to the x dimension. Specifically, four types of turns are disallowed: two in a clockwise cycle and two in a counterclockwise cycle. The basic concept behind the turn model is to prohibit a minimum number of turns and, hence, increase the routing adaptivity. In general, only one turn is prohibited in each cycle. For example, in a positive-first turn model two types of turns are disallowed (one for each cycle); that is, the turns from the negative to positive directions. The odd-even turn model restricts the locations where some of the turns can occur so that an EN (east-north) turn and a NW turn are not taken at nodes in the same column, and neither are an ES turn and a SW turn. Specifically, the odd-even turn model tries to prevent the formation of the *rightmost column segment of a cycle*. Chiu gave two rules for turn [7]:

Rule 1: *Any packet is* not *allowed to take an EN turn at any node located in an even column, and it is* not *allowed to take a NW turn at any node located in an odd column.*

Rule 2: *Any packet is* not *allowed to take an ES turn at any node located in an even column, and it is* not *allowed to take a SW turn at any node located in an odd column.*

Figure 2 shows these two rules on the EN, NW, ES, and SW turns. These four turns are called *sensitive turns*. Turns without restriction are called *insensitive turns*. A small triangle is placed at each sensitive turn that is permissible (as shown in Figure 2). Forbidden turns are represented as ones with dashed lines. A turn in an even (odd) column is represented by E (O). Basically in an odd-even turn model, *once east-bound starts, no more west-bound is allowed in the routing process*. Again, four directions are defined as: East ($+y$), South ($-x$), West ($-y$), and North ($+x$). (Unlike the convention the north-south axis is used here as the x axi to match the notation used in the odd-even turn model.)

To support a minimum and partially adaptive routing in a fault-free 2-D mesh, we consider a *restricted zig-zag routing* based on the odd-even turn model. The zig-zag routing represents an adaptive and minimal routing in 2-D meshes, where each hop in the routing process can be selected from one of the two profitable dimensions (i.e., the packet will get closer to the destination at each hop). The term “restricted” comes from the fact that a turn in zig-zag routing can only be made in a certain column. If a turn can only be made in an odd (even) column, it is called *even-(odd-)restricted zig-zag routing*. Routing can be divided into *EW-routing* (from east to west) and *WE-routing* (from west to east). The case for the offset along the y dimension (Δy) and the x dimension (Δx) being zero are excluded, since the packet can be routed straight to the destination without making any turn. Based on the rules in the odd-even turn model, EW-routing follows the odd-restricted zig-zag routing, unless the odd-column-turn is in the destination column. Figure 3 (a) shows an EW-routing where the destination is in the 2nd quadrant of the source, where label “E” represents an even column. For WE routing, if the destination is in an odd column, the even-restricted zig-zag routing is followed (see Figure 3 (b) for the 1st quadrant routing). If the destination is in an even column, the same rule as above is followed to reach the west neighbor d' of the destination d. The route completes with an east hop from d' to d (as shown in Figure 3 (c) for the 1st quadrant routing). The routing process in the 3rd and 4th quadrants is similar to the one in the 1st and 2nd quadrants, respectively.

3.3 Localized algorithms

In a *localized algorithm* [10, 27], which is a special type of decentralized algorithm, each processor (process) interacts with others in a restricted vicinity, but nevertheless collectively achieves a desired global objective. This type of algorithm is useful in a system with a set of independent, autonomous processors (processes). In general, each processor (process) performs exceedingly

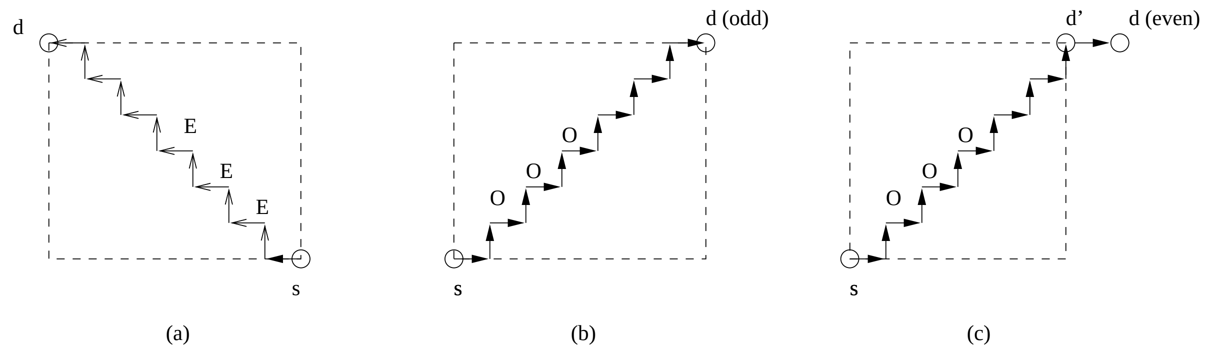

Figure 3. Minimum and partially adaptive routing in odd-even turn model: (a) EW-routing in the 2nd quadrant. (b) EW-routing in the 1st quadrant (destination in the odd column). (c) EW-routing in the 1st quadrant (destination in an even column).

simple tasks, such as maintaining and propagating information "markers". In this paper, we study several localized algorithms in which only neighbors exchange and update their markers.

3.4 Faulty blocks

We first introduce a special faulty block model. Faulty nodes in a 2-D mesh are contained in a set of disjointed rectangular faulty blocks. The *regular faulty block* model is defined as follows: *All nonfaulty nodes are safe initially. A nonfaulty node is changed to unsafe if it has two unsafe or faulty neighbors in different dimensions.* Figure 4 shows three sample faulty blocks where black nodes are faulty, gray are unsafe and other nodes are safe. The boundary rings are shown in boldface.

In the extended faulty block model proposed in this paper, each faulty block is surrounded by a boundary ring consisting of four boundary lines, one for each direction. The boundary line at the east (and west) side of the block consists of two lines. Two faulty blocks are disjointed if the boundary ring of one faulty block does not intersect with the other faulty block.

Definition 1: *All nonfaulty nodes are* safe *initially. A nonfaulty node is changed to* unsafe *if*

1. *it has two unsafe or faulty neighbors that are not both in the x dimension; or*

2. *it has an unsafe or faulty neighbor in the x dimension and an unsafe or faulty 2-hop neighbor*

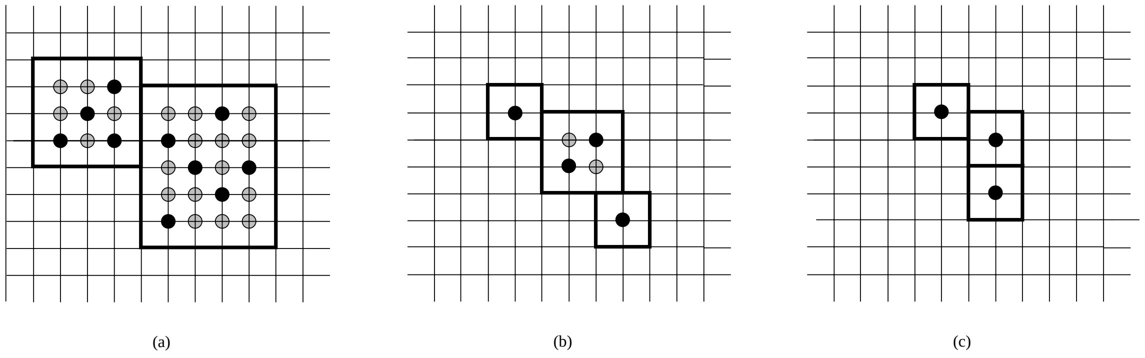

Figure 4. Three examples of faulty blocks where black nodes are faulty nodes and gray nodes are nonfaulty nodes.

(neighbor's neighbor) in the y dimension.

An extended faulty block consists of connected unsafe and faulty nodes. Note that although both unsafe and faulty nodes are included in faulty blocks, they are treated differently as will be seen later in the orthogonal faulty block model where certain unsafe nodes can be activated by removing them from the blocks. The difference between the extended faulty block definition (Definition 1) and the conventional one lies in the different treatments of adjacent nodes in different dimensions. An extended faulty block and its boundary nodes are so defined to facilitate fault-tolerant routing based on the odd-even turn model to be discussed in the next section. It can be easily shown that faulty blocks in 2-D meshes are disjointed rectangles. Let u: (u_x, u_y) and v: (v_x, v_y) be two nodes in a 2-D mesh, $d(u, v) = |u_x - v_x| + |u_y - u_y|$ denotes the distance between u and v. The *distance* between two faulty blocks A and B is defined as $d(A, B) = \min_{u \in A, v \in B}\{d(u, v)\}$. It can be easily shown that the distance between any two faulty blocks, A and B, is at least 3 along the y dimension or is at least 2 along the x dimension. Figure 5 shows three extended faulty blocks of the example in Figure 4. In the subsequent discussion, faulty blocks and extended faulty blocks are sometimes used interchangeably.

In the localized algorithm for safe/unsafe status (see Figure 6), each nonfaulty node is marked either safe or unsafe. Neighbors exchange and update their markers. Eventually, connected unsafe and faulty nodes form a faulty block (which is a global objective). To facilitate the decision process

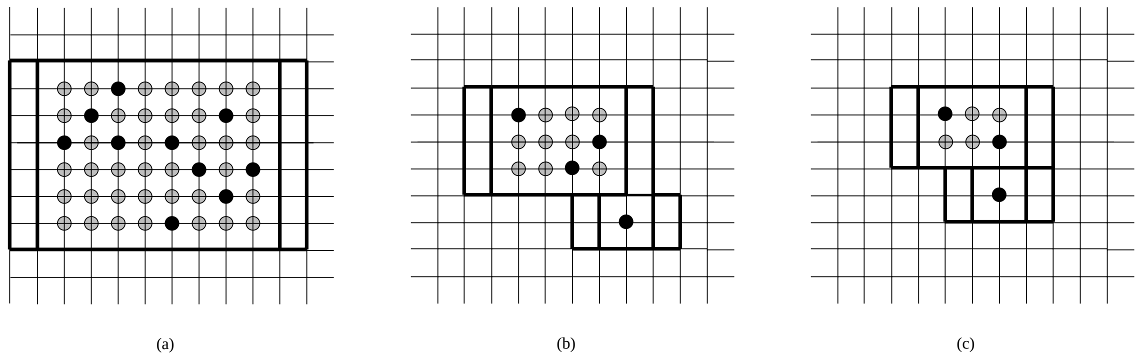

Figure 5. Three examples of extended faulty blocks in Figure 4.

of node status, each node sends its status to its 2-hop neighbors along the y dimension. It is assumed that each node knows the status of its neighbors.

Theorem 1: *Boundary nodes of a faulty block do not intersect with any other faulty block.*

Proof: Assume that node u is a boundary node of a faulty block A; that is, it is either a 1-hop neighbor along dimension x or a 1-hop or 2-hop neighbor along dimension y. Assume that node u also belongs to faulty block B. Based on the faulty block definition, faulty blocks A and B should be combined to form a single block. This brings a contradiction. ■

Although boundary nodes of a faulty block do not intersect with any other faulty block, boundary nodes of different faulty blocks may overlap; that is, a node can be a boundary node of more than one faulty block. The complexity of the safe/unsafe status procedure, in terms of the number of rounds needed, is the maximum diameter of faulty blocks in the mesh: $\max\{diam(A)\}$. Since the safe/unsafe process is a localized algorithm, a faulty block can be constructed in a decentralized way in a few rounds [26]. This property supports dynamic reconfiguration of faulty blocks.

4 Extended X-Y routing

We propose a routing process, called *extended X-Y routing*, which consists of two phases, similar to a regular X-Y routing. (That is why it still belongs to dimension-order routing although the other dimension is used within each phase to bypass faulty blocks encountered.) In phase 1, the offset along the x dimension is reduced to zero, and in phase 2, the offset along the y dimension is reduced

```
Safe/unsafe status:
1. All nonfaulty nodes are initialized to safe.
2. repeat
3.     doall
4.         (1) Nonfaulty node $u$ exchanges its status with its neighbors. In addition, the status
               of its east (west) neighbor is passed to its west (east) neighbor.
5.         (2) Change $u$'s status to unsafe if
6.                 (a) it has two unsafe or faulty neighbors that are not both in the $x$ dimension, or
7.                 (b) it has an unsafe or faulty neighbor along the $x$ dimension and an unsafe or
                       faulty 2-hop neighbor along the $y$ dimension.
8.     doall
9. until there is no status change
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Figure 6. A localized algorithm for determining safe/unsafe status.

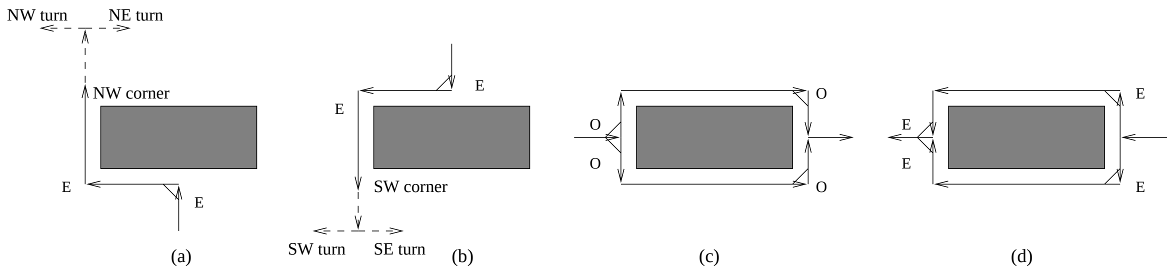


Figure 7. Two cases of routing along the x dimension (column) (a) north-bound and (b) south-bound and two cases along the y dimension (row) (c) east-bound and (d) west-bound.

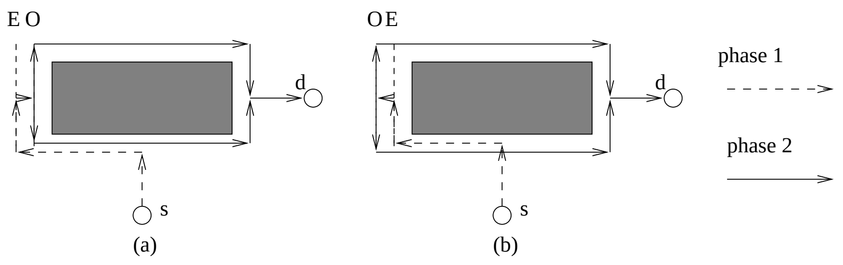


Figure 8. Two subcases of special routing.

to zero. Assume the source and destination nodes are both safe. Let $s : (s_x, s_y)$ and $d : (d_x, d_y)$ be the source and destination nodes, respectively. $\Delta_x = |d_x - s_x|$ and $\Delta_y = |d_y - s_y|$ are offsets along dimension x and dimension y, respectively.

The extended X-Y routing provides a special implementation of the requirement posed in the odd-even model and, at the same time, supports fault-tolerant routing. The extended X-Y routing (shown in Figure 9) follows the regular X-Y routing (and the packet is in a "normal" mode) until the packet reaches a boundary node of a faulty block (this block is called the *routing block*). At that point, the packet is routed around the block (and the packet is in an "abnormal" mode) clockwise or counterclockwise based on certain rules: Unlike routing in a fault-free routing, the fault-tolerant routing protocol has to prepare for "unforeseen" situations: a faulty block encountered during the routing process. The solution is to route around the block without using any forbidden turns. This is done by three means: (a) the packet should reside in an even column when reaching a north or south boundary node of the routing block in phase 1. (b) In phase 1, the packet should be routed around the west side, since once the packet is east-bound it cannot be changed to west-bound later. (c) The two boundary lines, one even and one odd, offer just enough flexibility for the packet to make turns for all situations. More specifically, during phase 1 the packet is routed around the routing block through the west side of the block. Even columns are used to route the packet along the x dimension (column). If the source is in an odd column, the first hop is to the west neighbor of the source (which is in an even column). In phase 2, to route around the routing block, odd columns (even columns) are used to perform routing along the y dimension when the packet is east-bound (west-bound) (see Figures 7 (c) and (d)). The packet is routed around the routing block either clockwise or counterclockwise in phase 2 (see Figures 7 (c) and (d)). Note that during the normal

Extended X-Y routing:

1. /* `the packet is sent to an even column first.` */

   (a) If the source is in an odd column and Δ_x is non-zero, then the packet is sent to its west neighbor in an even column.

2. /* `phase 1:  reduce` Δ_x`, the offset in the` x `dimension` */

   (a) (Normal mode) reduce Δ_x to zero by sending the packet north (or south) (with no 180° turn).

   (b) (Abnormal mode) when a north-bound (south-bound) packet reaches a boundary node of a faulty block, it is routed around the block clockwise (counter-clockwise) by following the boundary ring of the faulty block as shown in Figure 7 (a) (Figure 7 (b)). The packet takes the first even column turn whenever possible and step (a) is followed.

3. /* `phase 2:  reduce` Δ_y`, the offset in the` y `dimension` */

   (a) Once Δ_x is reduced to zero, a NW or NE turn is performed for the north-bound packet (see Figure 7 (a)) and a SW or SE turn is performed for a south-bound packet (see Figure 7 (b)). The selection of a turn depends on the relative location of the destination to the current node.

   (b) (Normal mode) reduce Δ_y to zero by sending the packet east (west) (with no 180° turn).

   (c) (Abnormal mode) when a east-bound (west-bound) packet reaches a boundary node of a faulty block, it is routed around the block, clockwise or counterwise, along odd columns of the boundary ring as shown in Figure 7 (c) (even columns of the boundary ring as shown in Figure 7 (d)). Routing around the block is completed when Δ_x is again reduced to zero and step (b) is followed.

Figure 9. Extended X-Y routing.

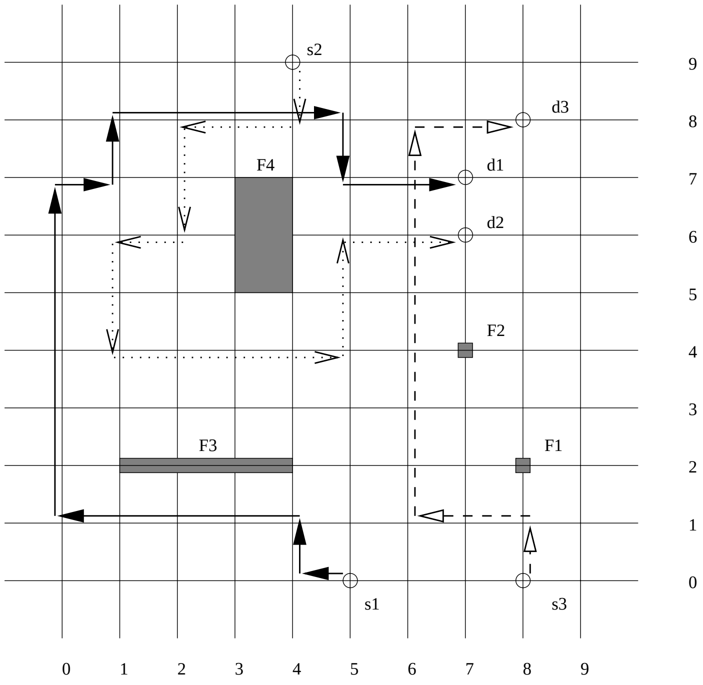


Figure 10. Three routing examples in a 10 $\times$ 10 mesh with four faulty blocks.

mode of routing the packet along the x or y dimension, no 180° turn is allowed. For example, the positive x direction cannot be changed to the negative x direction.

A special case occurs when the destination is at the east side of the routing block. In this case, when phase 1 completes, the routing packet is still at the west side of the routing block as shown in Figure 8. The even (marked as E) boundary column of the routing block is switched to the odd (marked as O) boundary column (two subcases are shown in Figures 8 (a) and (b)), and then, the packet is routed around the block either clockwise or counterclockwise. Figure 10 shows three routing examples (s_i, d_i), with $i \in \{1, 2, 3\}$, in a 10 $\times$ 10 mesh with four faulty blocks F_1, F_2, F_3, and F_4. Note that when the routing packet reaches a northwest (NW) or southwest (SW) corner of a routing block in phase 1, the packet goes straight, north-bound or south-bound, without further routing around the block (see Figures 7 (a) and (b)).

A fault-tolerant routing process is livelock-free if it can deliver packets from the source to destination, regardless of the number and location of faults. The following result shows that the extended X-Y routing is both deadlock-free and livelock-free in a 2-D mesh where faults are contained in a set of disjointed faulty blocks.

Theorem 2: *The extended X-Y routing is deadlock-free and livelock-free.*

Proof: In the routing process, all sensitive turns are permissible, based on the results from the odd-even turn model [7], the extended X-Y routing is deadlock-free. To show the livelock-free property, we only need to show that Δ_x (Δ_y) is eventually reduced to zero in phase 1 (phase 2). In phase 1, Δ_x is always reduced by one at each step (with no 180° turn), except when the packet is routed around a faulty block by going west. There are two cases of west-bound hops, one case is in the first hop for the packet to reach an even column and the other case occurs when the packet is routed around a faulty block. Since the size of each faulty block is limited, the number of west-bound hops is a finite number. In addition, although the packet may have to route around several faulty blocks (or several abnormal modes), Δ_x is reduced by at least one in a normal mode between every two adjacent abnormal modes. Therefore, Δ_x is reduced to zero in a finite number of steps in phase 1. Similarly, Δ_y is also reduced by one at each step in phase 2, except when the packet is routed around a faulty block. Since the size of each faulty block is limited, using the same argument used in phase 1, Δ_y is eventually reduced to zero in a finite number of steps in phase 2. Note that when a packet is routed around a faulty block in phase 2, Δ_x may temporarily become non-zero, based on the routing process, Δ_x is reduced to zero again when the process of routing around the faulty block is completed. ■

Note that the destination is not a boundary node of any faulty block. This is to prevent the following case: If the destination is at the east side of a faulty block and it is on an even boundary line which is closer to the block than the odd boundary line, then the rightmost column segment of a cycle may be constructed when an east-bound packet routes around the block as shown in Figure 7 (c). However, this restriction can be removed by introducing two virtual channels as will be shown in Section 6.

5 Orthogonal Faulty Blocks

A faulty block may include many nonfaulty nodes labeled as unsafe as shown in Figure 5. Many unsafe nodes can be activated and removed from a faulty block while still keeping its convexity. The following definition provides such a special convex fault region.

Definition 2 [26]: *A region is* orthogonal convex *if and only if the following condition holds: For* *any horizontal or vertical line, if two nodes on the line are inside the region, then all the nodes on the line that are between these two nodes are also inside the region.*

A fault region that is orthogonal convex is called an *orthogonal faulty block*. In the following, we propose a simple decentralized formation of orthogonal faulty blocks from a given set of faulty blocks. Given a faulty block, the corresponding orthogonal faulty block(s) can be derived by assigning *enabled/disabled* status to safe/unsafe nodes in the faulty block.

Definition 3 [26]: *All safe nodes are marked* enabled*. An unsafe node is initially marked* disabled*. It is changed to the* enabled *status if it has two or more enabled neighbors.*

An orthogonal faulty block consists of connected disabled and faulty nodes. Wu [26] showed that a fault region derived from the enabled/disabled process is an orthogonal convex polygon. In addition, each region is the *smallest orthogonal convex polygon that covers all the faulty nodes within the region*.

Figure 12 shows the corresponding orthogonal faulty blocks for three extended faulty blocks in Figure 5. White nodes are unsafe but enabled nodes. Gray nodes are unsafe and disabled nodes. Black nodes are faulty nodes. Safe nodes are not shown. In fact, the enabled/disabled status replaces the safe/unsafe status. That is, a nonfaulty node is labeled either enabled or disabled in orthogonal faulty blocks. For the faulty block in Figure 5 (a), the corresponding orthogonal faulty block is substantially reduced. The two faulty blocks in Figure 5 (b) are partitioned into three orthogonal faulty blocks as shown in Figure 12 (b). A localized algorithm for determining enabled/disabled status is given in Figure 11.

However, if boundary nodes of an orthogonal faulty block are defined the same way as in a regular faulty block, two boundary lines that are at the east and west sides of the block may not exist. For example, the two faulty blocks in Figure 5 (c) are partitioned into three orthogonal faulty blocks but two of them, A and B, do not meet the boundary node condition (Theorem 1). The problem in Figure 12 (c) is that unsafe nodes between two faulty nodes along the y dimension (and both faulty nodes are either in the same row or two adjacent rows) should not be enabled to ensure that boundary nodes of an orthogonal faulty block do not intersect with another block. Since such a node "connects" two adjacent orthogonal faulty blocks, it is simply called a *connector*.

In the extended disabled/enabled status, we first identify the disabled/enabled status for each

```
Enabled/disabled status:
1.  All unsafe nodes are initialized to disabled.
2.  All safe nodes are enabled.
3.  repeat
4.      doall
5.          (1) Unsafe node $u$ exchanges its status with ones of its neighbors.
6.          (2) Change $u$'s status to enabled if it has two or more enabled neighbors.
7.      doall
8.  until there is no status change
```

Figure 11. A localized algorithm for determining enabled/disabled status.

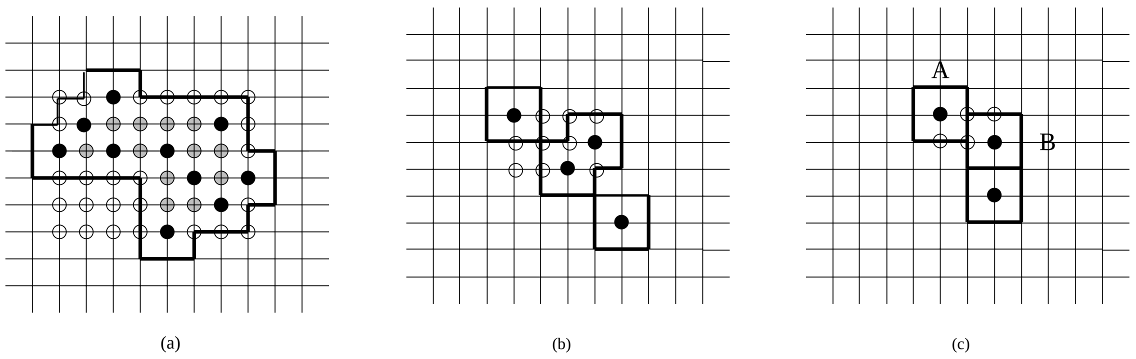


Figure 12. The orthogonal faulty blocks of three extended faulty blocks in Figure 5.

nonfaulty node like in the orthogonal faulty block. Then each connector is explicitly disabled, even if it has been enabled early. Also, in order to distinguish between different types of nodes, the disabled/enabled status does not supersede the safe/unsafe status, instead it is used as an additional status indicator. Therefore, a nonfaulty node has one of the three status: (safe, enabled), (unsafe, enabled), or (unsafe, disabled). In the following, we provide a formal definition for a connector.

Definition 4: *An unsafe node is called a* connector *if one of its east and west neighbors is faulty and the other is either faulty, or, nonfaulty with a faulty north or south neighbor.*

An *extended orthogonal faulty block* is an orthogonal faulty block with all connectors disabled.

```
Extended enabled/disabled status:
1.  Call procedure enabled/disabled status but still keeping the safe/unsafe status of each node.
2.  doall
3.      (1) Each unsafe node with a faulty south or north neighbor is called quasi-faulty.
4.      (2) Each unsafe node is a connector if one of its east and west neighbors is faulty and the
            other is either faulty or quasi-faulty.
5.      (3) Each connector is explicitly disabled.
6.  doall
```

Figure 13. A localized algorithm for determining extended enabled/disabled status.

A localized algorithm for extended enabled/disabled status is given Figure 13. Note that "quasi-faulty" in the extended enabled/disabled status process is not a final status label, and it is introduced just to identify connectors. Since an extended orthogonal faulty block is generated from a given faulty block, the complexity of extended enabled/disabled status procedure is still the maximum diameter of faulty blocks. In the subsequent discussion, extended orthogonal faulty blocks and orthogonal faulty blocks are sometimes used interchangeably. Figure 14 shows the result of applying the extended enabled/disabled process to the examples in Figure 5. The nodes with a cross are connectors.

Preposition 1: *A fault region derived from the extended enabled/disabled process is an orthogonal convex polygon.*

Preposition 2: *Any boundary node of an orthogonal faulty block derived from the extended enabled/disabled process does not belong to any other orthogonal convex polygon.*

The proofs of Propositions 1 and 2 follow directly the ones used in [26] for properties of regular orthogonal faulty blocks. Note that the extended orthogonal faulty block is no longer the smallest convex region covering all faults in the region. However, the marking process can be enhanced to reduce the block size. For example, the connector definition (Definition 4) can be changed to the following: *An unsafe node is called a connector if its east neighbor is faulty and its west neighbor is either faulty, or, nonfaulty but with a faulty north or south neighbor.* With this change, two

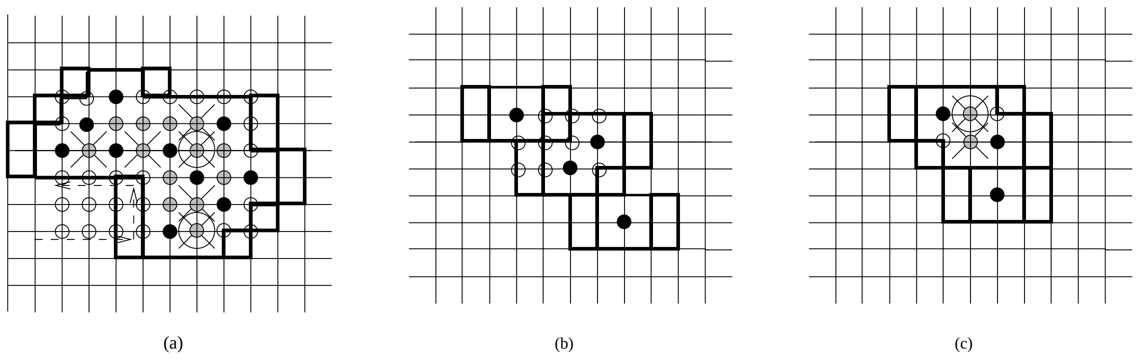


Figure 14. The extended orthogonal faulty blocks of three extended faulty blocks in Figure 5.

connectors with double circles in Figure 14 (a) are no longer connectors: one is still disabled and the other at the bottom row enabled based on the enabled/disabled status process. In Figure 14 (c), the doubly-circled connector is enabled.

To see the difference between regular faulty blocks (FB) (orthogonal faulty blocks (OFB) (Definition 3)) and extended faulty blocks (Definition 1) (extended OFB (Definition 4)) in terms of the number of nonfaulty nodes included in these blocks, we conducted a simulation study on a 100×100 mesh where a number of faults are randomly generated. Figure 16 (a) shows the numbers of nodes covered in FBs and extended FBs for given numbers of faults. Figure 16 (b) shows the numbers of nodes covered in OFBs and extended OFBs for given numbers of faults. It is clear from the results that both orthogonal faulty blocks and extended orthogonal faulty blocks enable some nonfaulty nodes from faulty blocks and extended faulty blocks, respectively. The number of unsafe but enabled nodes is not significant, because we used a random fault distribution and the faulty blocks generated tend to be small. We expect better results if faults are "clustered". Also, we observe that the two-boundary-line requirement in the extended models "inflates" the overall size of blocks. However, such inflation is not significant when the number of faults is relatively small.

To apply the extended X-Y routing in 2-D meshes with orthogonal faulty blocks, the source should be an enabled node and the destination should be a safe node that is not a boundary node of an orthogonal faulty block. In this case, an unsafe but enabled node (a node inside the extended faulty block but outside the extended orthogonal faulty block) can be a source but not a destination.

Also, such a node can be used for routing like a “lamb” node in [15].

The proposed extended X-Y routing can be easily extended to 2-D meshes with orthogonal faulty blocks. If a routing block is orthogonal, care should be taken in phase 2 when a routing packet goes around the block. The selection of either the clockwise or counterclockwise direction to route around a fault region becomes important (and is no longer arbitrary). The formation of the rightmost column of a cycle may occur as shown in Figure 14 (a) when an east-bound packet goes around the block clockwise (with respect to the block). To avoid this situation, the boundary lines around east and west sides of an orthogonal faulty block are associated with *directional information* as shown in Figure 15 (a). Basically, boundary nodes at the west (east) side of an orthogonal faulty block should “point toward” east (west).

A distributed formation of directional information together with boundary nodes of an orthogonal faulty block using a localized algorithm is shown in [25]. Here we provide another simple implementation. Let $\min_x$ and $\max_x$ ($\min_y$ and $\max_y$) be the minimum and maximum coordinates of the original faulty block of an orthogonal faulty block A along dimension x (dimension y), respectively. The rectangle spanning from node $(\min_x, \min_y)$ to node $(\max_x, \max_y)$ is called a *container* for block A. A *west-most boundary node* is a west boundary node of a faulty block(s) (constructed from the safe/unsafe status process) that maintains its column position after the formation of the orthogonal faulty block (constructed from the extended enabled/disabled status process). Such a node distributes its row position (r) to all west boundary nodes. A packet that reaches a west boundary node with a higher row position than r will route around the block clockwise; otherwise, the packet will route around the block counterclockwise. Node X (at the edge of the container) in Figure 15 corresponds to a west-most boundary node. When several west-most boundary nodes initiate the construction of directed boundary lines at the same time, their row numbers can be used to break a tie.

Once directed boundary lines are defined, the extended X-Y routing can be directly applied. The only change occurs in phase 2, the routing packet has to follow the directed boundary lines when routing around a routing block. In phase 1, the packet still goes west-bound around a routing block with one minor, but subtle, change: When the packet reaches a northeast (NE) or southeast (SE) corner (see Figure 15 (a)) of a routing block, the packet should be sent west immediately. Figures 15 (b) and (c) show two routing examples, where routing blocks are orthogonal faulty

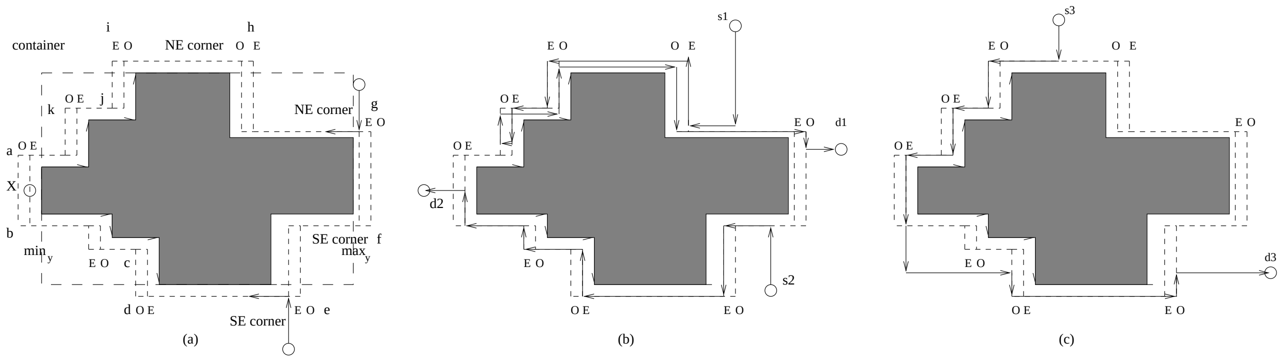
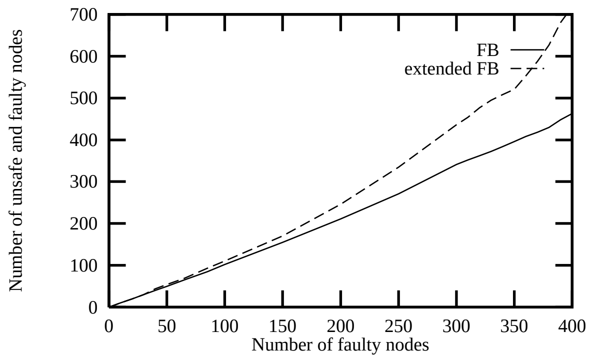


Figure 15. Routing along orthogonal faulty blocks.

blocks, and s_i and d_i represent source and destination, respectively. In phase 1, routing around a routing block completes when either the packet reaches the leftmost boundary line or Δ_x is reduced to zero. In the former case (as shown in Figure 15 (c)), the packet is then routed along dimension x until Δ_x is reduced to zero. In phase 2, routing around a routing block completes when either the packet reaches the rightmost boundary line or Δ_x is reduced to zero. In the former case, the packet is routed along dimension x until Δ_x is reduced to zero, and then Δ_y is reduced by sending the packet along dimension y. In the latter case, the packet is routed directly along dimension y to reduce Δ_y (as shown in the example (s_1, d_1) of Figure 15 (b)).

Theorem 3: *Using the orthogonal faulty block model, the modified extended X-Y routing is still deadlock-free and livelock-free.*

Proof (Sketch): In phase 1, assume that the packet is north-bound (the south-bound case can be treated in a similar way), routing around a routing block involves a sequence of the following turns: NW, (WS, SW)*, (WN, NW)*, WN, where (WS, SW)* represents zero or more repetitions of WS, SW turns (as shown in the example (s_2, d_2) of Figure 15 (b)). All sensitive turns, NW and SW, occur in even columns and they are permissible. In the transition between phase 1 and phase 2, either a NW or NE is performed in an even column. In phase 2, assume that the packet is east-bound (the west-bound case can be treated in a similar way), routing around a routing block (if any) involves a sequence of the following turns if the packet is routed in the counter-clockwise direction: ES, (SE, ES)*, SE, (EN, NE)*, EN. If the packet is routed in the clockwise direction, the following sequence of turns is used: EN, (NE, EN)*, NE, (ES, SE)*, ES. All sensitive turns



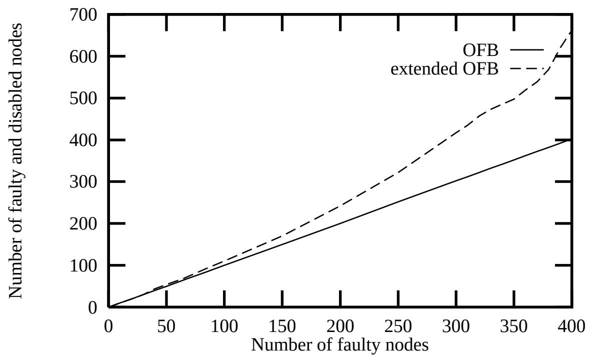


Figure 16. The numbers of nodes covered in (a) FBs and extended FBs and (b) OFBs and extended OFBs.

are performed in odd columns and they are permissible. Once Δ_x is reduced to zero, either a SE or NE turn is performed in an odd column to reduce Δ_y. Because the destination is outside a container, the packet is still east-bound after phase 2. Hence, the modified extended X-Y routing is still deadlock-free and livelock-free. ■

6 Extensions

In this section, we provide some ideas for extensions, which include partial adaptive routing, traffic- and adaptivity-balanced routing using virtual networks, and removing constraints using virtual channels and networks.

6.1 Partial adaptive routing

The extended X-Y routing is deterministic, that is, there is only one routing path (except when a packet routes around a routing block). In the following, we propose a partially adaptive routing based on the restricted zig-zag routing. Again, routing can be divided into *EW-routing* (from east to west) and *WE-routing* (from west to east). WE-routing follows the extended X-Y routing which consists of phase 1 and phase 2 as discussed in the previous section (see Figures 17 (a) and (b)). EW-routing follows the odd-restricted zig-zag routing which consists of a sequence of alternating phase 1 and phase 2 (see Figures 17 (c) and (d)). WE-routing is still deterministic while EW-

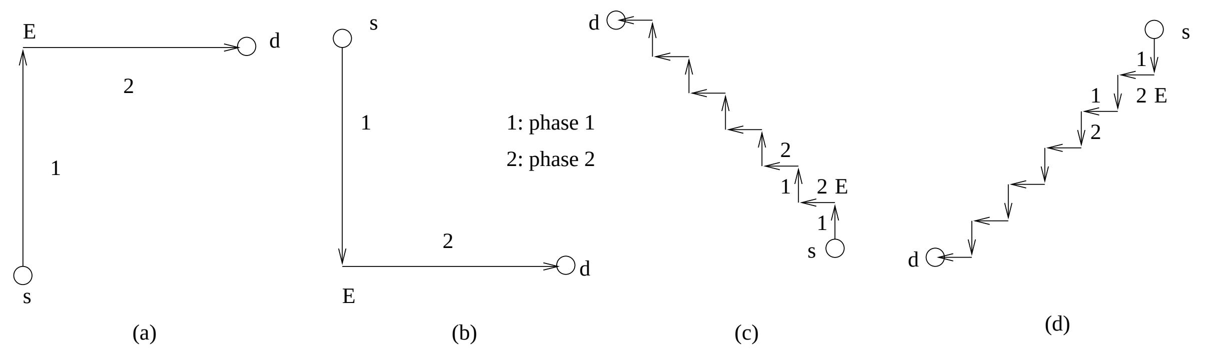


Figure 17. Partial adaptive routing: (a) and (b) extended X-Y routing for WE-routing and (c) and (d) restricted zig-zag routing for EW-routing.

routing is partially adaptive. The reason that WE-routing has to follow deterministic routing is the following: Suppose WE-routing is allowed to use the restricted zig-zag routing; that is, the packet can be east-bound before phase 1 completes. If a faulty block is reached before phase 1 completes, the packet is forced to route around the block by going west. This will violate the restriction of the odd-even turn model: once east-bound starts, no more west-bound is allowed in the routing process.

Let Δ_x^1 and Δ_y^1 (Δ_x^2 and Δ_y^2) be the offsets along dimensions x and y, respectively, in phase 1 (phase 2). The requirement at each phase is the following: In phase 1, Δ_x^1 is monotonically decreasing, and in phase 2, Δ_y^2 is monotonically decreasing and Δ_x^2 remains unchanged. It can be easily shown that as long as the above requirement is met in each phase, the restricted zig-zag routing is livelock-free. The freedom of deadlock of the restricted zig-zag routing is obvious, since all turns in both phase 1 and phase 2 are permissible.

6.2 Traffic- and adaptivity-balanced routing using virtual networks

The proposed routing protocol does not make use of resources (channels) evenly. It is obvious that even columns are heavily used in routing in the x dimension. To balance the channel usage, we can use two versions of the routing protocol. One version is based on Rules 1 and 2 and heavily uses even columns, and the other one is discussed below and heavily uses odd columns. In the second version of the extended odd-even turn model, still the rightmost column segment of a cycle

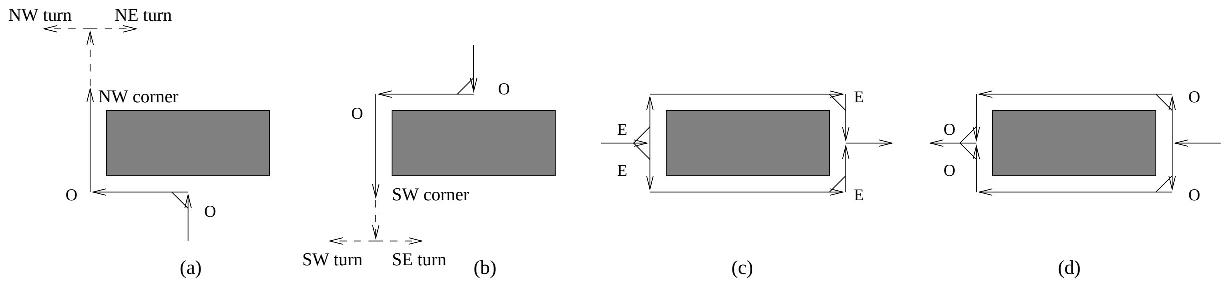


Figure 18. Two cases of phase 1 routing (a) and (b) and two cases of phase 2 routing (c) and (d) using Rules 1$^{'}$ and 2$^{'}$.

is prevented. However, *the rule of even and odd is exchanged*.

Rule $1^{'}$: *Any packet is* not *allowed to take an EN turn at any node located in an odd column, and it is* not *allowed to take a NW turn at any node located in an even column.*

Rule $2^{'}$: *Any packet is* not *allowed to take an ES turn at any node located in an odd column, and it is* not *allowed to take a SW turn at any node located in an even column.*

Figure 18 shows the permissible EN, NW, ES, and SW turns under Rules $1^{'}$ and $2^{'}$ in phases 1 and 2. In the second version of the extended odd-even turn model, odd columns are used to route the packet in the x dimension (in phase 1). The role of even and odd is also exchanged when routing around faulty blocks in phase 2. To support two versions of the extended odd-even turn model, two virtual networks, VN_1 and VN_2, are used. VN_1 is used to enforce Rules 1 and 2 while VN_2 is applied to implement Rules $1^{'}$ and $2^{'}$. Each virtual network may support several virtual channels. Figure 19 shows the notions of virtual channels and virtual networks using a 2×2 mesh (Figure 19 (a)). Figure 19 (b) shows a 2×2 mesh with two virtual channels, VC_1 and VC_2, but still one network. Figure 19 (c) shows a 2×2 mesh with two virtual networks VN_1 and VN_2. VN_1 (and VN_2) consists of virtual channels VC_1's (VC_2's) only.

We use the configuration shown in Figure 19 (c) for implementation. A virtual network is selected whenever a packet is injected into the network. Each packet stays in the virtual network until it reaches the destination. It is possible to allow switching from VN_1 to VN_2 (see Figure 19 (c)) during the routing process to increase adaptivity without causing deadlock. We adopt the following rule for a source to select a virtual network: VN_1 *is used if the destination is in an odd*

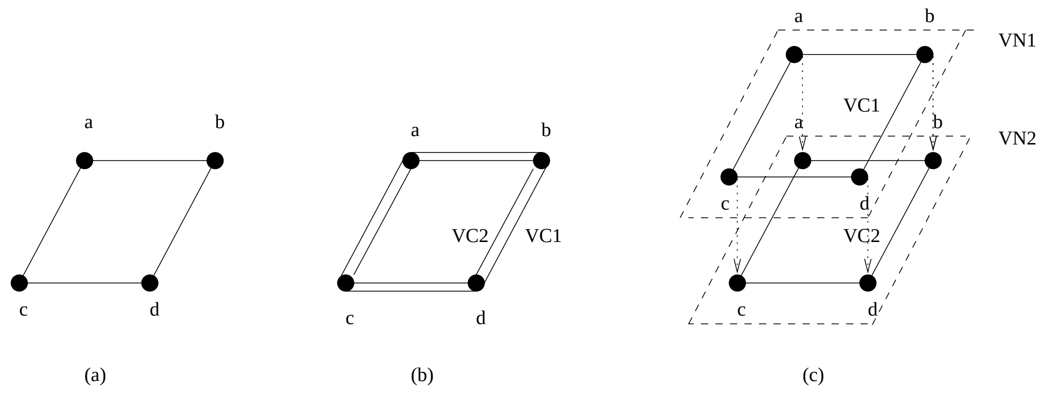


Figure 19. (a) A 2×2 mesh, (b) a 2×2 mesh with two virtual channels VC_1 and VC_2, and (c) a 2×2 mesh with two virtual networks VN_1 and VN_2.

column; otherwise, VN_2 is chosen. The only exception is when the source is in the 0 column. Since the 0 column does not have a west neighbor in an odd column, VN_1 is used even though the destination may be in an even column.

Note that other versions of the extended odd-even turn model can be derived either by *preventing the leftmost column segment of a cycle* or by *exchanging the role of column and row*. Rules preventing the leftmost column segment of a cycle not only provide a traffic-balanced routing to complement Rules 1 and 2 but also provide an adaptivity-balanced routing when used together with Rules 1 and 2. Under these new rules, WE-routing should adopt the extended X-Y routing while EW-routing should follow the restricted zig-zag routing. When two versions are used together, they provide an adaptivity-balanced routing between EW-routing and WE-routing.

6.3 Removing constraints using virtual channels and virtual networks

So far we focus on presenting the basic idea without going into the messy details of boundary situations. These situations include faulty nodes at edges (or adjacent to edges) of the 2-D mesh and destinations adjacent to a faulty block. Many existing approaches [3, 23] can be applied to handle the former case where virtual channels are used to route around faulty blocks at the edges of the mesh. Note that when the proposed approach is applied to the 2-D torus network with wraparound connections, there will be no fault constraint at the edge of the network! However, nodes around each dimension form a ring. Virtual channels (or virtual networks) need to be introduced to remove potential cyclic dependency.

Here we focus on removing the condition (3) which requires the destination not to be a boundary node of any faulty block. In fact, the proposed algorithm works for all destinations that are boundary nodes of faulty block, except ones at the east side of a faulty block. Refer to Figure 4 (c) where an ES turn is made in an odd boundary line. If the corresponding even boundary line is at the west of the odd boundary line (i.e., closer to the faulty block) and the destination is in the even boundary line, it will force a SW turn in the odd boundary line (which is not permissible) as the last hop. To handle this situation, two virtual channels VC_1 and VC_2 are used in those even boundary lines that are adjacent to faulty blocks. All hops use VC_1's except the last hop which is a SW turn in the odd boundary line.

Two virtual networks provide an even simpler solution. This approach not only balances traffic but also increases the scope of applicability; that is, the constraint that the destination is not a boundary node of a faulty block can be removed. Rules 1 and 2 are implemented using VN_1, and it will take care of all destinations in odd columns. In this case, no SW or NW turn is needed at the east side of a faulty block. Rules $1'$ and $2'$ are implemented using VN_2, and it handles all destinations in even columns. When the source is in the 0 column and uses VN_1, the packet can still be switched to VN_2 if needed when the destination is in an even column.

The virtual channel approach can also be applied to the orthogonal faulty block model. Recall the additional constraint on the orthogonal faulty block model: The destination must be outside the container of any orthogonal faulty block. That is, unsafe but enabled nodes (i.e., nodes inside the container but outside the orthogonal faulty block) are used only as sources or intermediate nodes to bypass traffic, but not destinations. In fact, unsafe but enabled nodes in a container $[min_x : max_x, min_y : max_y]$ form up to four *connected components*. The one containing node (min_x, max_y) is called SE section (see Figure 20), the one containing node (min_x, min_y) SW section, the one containing node (max_x, max_y) NE section, and the one containing node (max_x, min_y) NW section. In fact, destinations at SW and NW sections are allowed without causing any problem. The problem occurs when the packet routes around the east side of a faulty block through a sequence of ES and SE turns in odd columns. If the destination is inside the SE section, a SW turn in an odd column cannot be avoided (as shown in Figure 20 for a phase 2 routing). A sim-

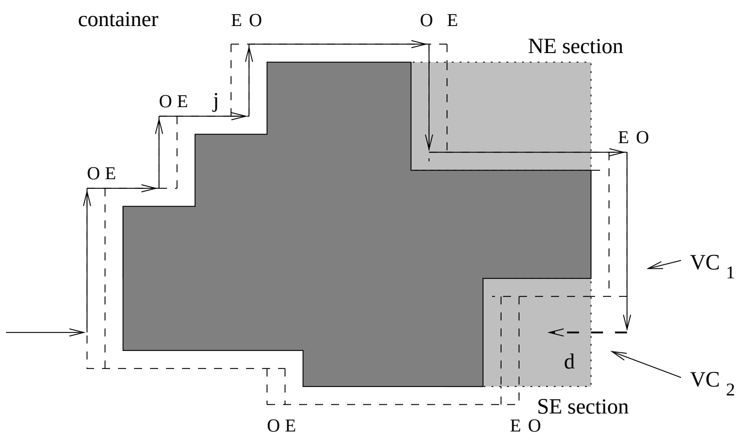


Figure 20. A SW turn in an odd column that switches from VC_1 to VC_2.

ilar situation occurs when the packet routes around the east side of the block through a sequence of EN and NE turns in odd columns and the destination is inside the NE section. The solution is again using two virtual channels: VC_1's are used throughout until a SW turn (NW turn) in an odd column is made. In this case, the packet enters the SE section (NE section) of the faulty block. In the remaining steps VC_2's are used until reaching the destination. It is easy to show that the SE or NE section is fault-free. Therefore, the remaining steps can be completed without switching virtual channels.

To see the difference between extended faulty block (FB) model (Definition 1) and extended orthogonal faulty block (OFB) model (Definition 4) in terms of the number of detours (i.e., extra hops) generated for a routing process, we conducted a simulation study on a 100×100 mesh where a number of faults are randomly generated. The location of the destination is outside any extended FB in both the extended FB and extended OFB models. In the extended OBF* model, the location of the destination can be inside an extended FB, but still outside any extended OFB (see the lightly shadowed area in Figure 20). The average number (Figure 21 (a) (and percentage (Figure 21 (b)) of detours for three cases are very close for the number of faults ranging from 1 to 400. Still, extended OFB stays very close to extended FB. That is, the irregular shape of OFB does not cause too many detours as long as the number of faults is not very large. The extended OFB* also stays close to extended FB when the number of faults is relatively small. However, the number of detours increases sharply when the number of faults is large. This is because unsafe but

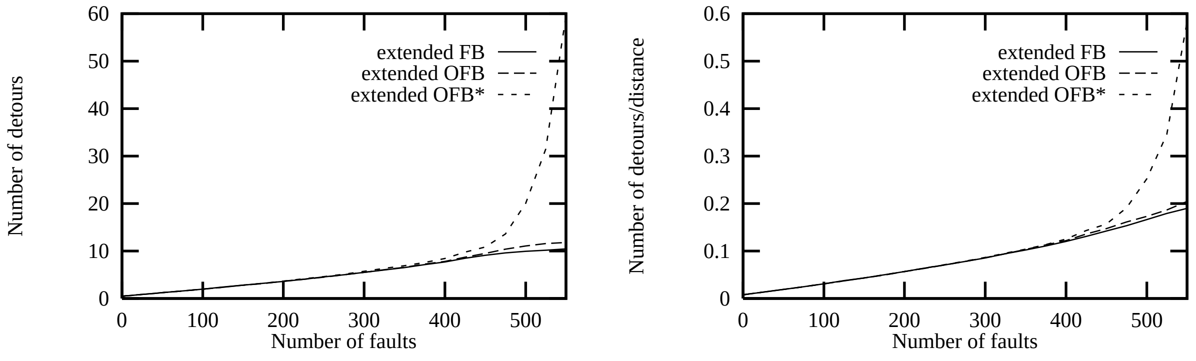


Figure 21. (a) The average number of detours and (b) The ratio of average number of detours and average distance between the source and destination nodes.

enabled nodes (i.e., more nodes in NE section and SE section of the block as shown in Figure 20) become dominant as faults increase, and the number of detours increases dramatically.

When the extended X-Y routing is used in a fault-free 2-D mesh, two extra steps will be introduced if the source is in an odd column and the destination is at the east side of the source (i.e., WE-routing). This situation occurs with a probability of 0.25 under a random distribution of the source and destination nodes. No extra hop will be introduced (i.e., the route is minimum) for all other cases. Two virtual networks (or virtual channels) can be used to eliminate extra hops in fault-free 2-D meshes: When the source is in an even column, VN_1 is used following Rules 1 and 2; otherwise, VN_2 is used following Rules $1'$ and $2'$.

7 Conclusions

In this paper, we have proposed a simple and efficient fault-tolerant and deadlock-free routing in 2-D meshes without virtual channels. This approach is based on the popular X-Y routing and the odd-even turn model. The novelty of the approach is the use of two boundary lines at the east and west of a faulty block. This gives just enough flexibility to route around a faulty block, and at the same time, to avoid certain turns that may cause deadlock. The proposed approach can be applied to 2-D meshes with any convex type of faulty blocks with simple modification. The approach can also be easily extended to support partially adaptive routing using a restricted zig-zag routing and traffic-balanced routing using two virtual channels and networks. We have shown the use of localized algorithms to construct rectangular faulty blocks, a special type of convex faulty blocks, and boundary lines. Our future work includes applying the extended odd-even turn model to high dimensional meshes and extension to handle dynamic faults during the routing process.

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