Some Analytical Results on a Localized Pruning Method for Connected Dominating Sets in MANETs

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Abstract

Generating a small size connected dominating set (CDS) for message routing in wireless ad hoc networks is always a challenging problem. In a recent paper, a local pruning algorithm called restricted rule-k has been proposed, and succeeds in generating a small size CDS. In this paper, a statistical analysis on the size of the CDS generated is presented. For a network of N nodes (where $N \to \infty$) that are uniformly randomly generated in a square of size $L_N \times L_N$ (where $L_N \to \infty$), three results are obtained. (1) It is proved that the node degree distribution of such a network follows a Poisson distribution. (2) The expected size of a CDS that is derived by the restricted pruning rule-k is a decreasing function with respect to the node density $\hat{n}$. For $\hat{n} \geq 30$, it is found that the expected size is $N/\hat{n}$. (3) A tighter lower bound on the expected size of a CDS, for Poissonian node degree distribution, is deduced. For a network of node denisty $(\lambda + 1)$, the lower bound is $\left\{\frac{1}{\lambda} - \frac{\lambda+1}{\lambda}\exp(-\lambda)\right\} N$ which is larger than the lower bound recently deduced by Hansen et al..

Keywords: Connected dominating sets (CDS), mobile ad hoc networks (MANET), restricted pruning rule, wireless networks

1 Introduction

In wireless ad hoc networks, selection of a set of nodes for efficient message routing is always a crucial problem. Much research has been done in the last decade aiming to find an efficient and simple method that can select such nodes. Amongst them, one promising distributed algorithm [11] is based on the idea of connected dominating set (CDS) [4]. Simulation and theoretical studies have shown that this CDS based algorithm is able to generate a small node set, responsible for message routing, without adding much computational overhead on the network [2, 11]. A CDS is a subset of nodes in which every node not in this subset is adjacent to at least one node in the subset. Via a CDS, each node can be ensured to have at least one path to send or receive messages from other nodes within the network. Basically, construction of a CDS is not difficult. The real challenge is how to construct a minimal CDS. It is an NP-complete problem [1].

Many approximation algorithms have been proposed in the past few years in order to construct a small CDS. Basically, those algorithms can be classified into centralized and decentralized algorithms. For a centralized algorithm [3, 4], a computationally powerful central host is normally assumed. It is responsible for running a construction algorithm based on global information and then broadcasting the results to the nodes in the network. As one can imagine that the volume of global information could be huge when the total number of nodes is large, and the overhead for gathering information could make the algorithm unsuitable for on-line implementation.

Decentralized algorithms [2, 6, 7, 8, 10, 11], on the other hand, require only local information — the IDs of direct neighbors and the neighbors of neighbors. The overhead for information gathering is minimal. Each node simply determines whether it is a node of the CDS in accordance with these local information. Thus, no central host is required.

A simple and yet efficient decentralized algorithm has been proposed by Wu & Li in [11] (and later extended by Dai & Wu in [2]) to tackle such a problem. In their algorithms, a CDS is constructed by going through two processes, namely the marking process and the pruning process. After exchanging information

with its direct neighbors, each node runs a marking step. in which a node will mark itself *true* if it has two unconnected neighbors. Otherwise, it will mark itself *false*. Once the marking process has finished, each *true* node runs a pruning step. Each marked node checks to see if its local condition fulfils the conditions specified by a pruning rule-k. In accordance with pruning rule-k, a marked node unmarks itself if there exists a set of connected nodes whose coverage can cover all its neighbors and at the same time the ID of the marked node is smaller than the IDs of those connected nodes. If the connected nodes are restricted to direct neighbors of the marked node, the pruning rule is called restricted rule-k.[1] Otherwise, it is called unrestricted rule.

Although the Wu & Li decentralized algorithm is simple and yet efficient in terms of computational complexity, only Dai & Wu in [2] and Hansen *et al* in [5] have provided analytical studies on the size of a CDS derived by such an algorithm. Let N be the total number of nodes and $N \to \infty$, Dai & Wu showed that the size of a pruning rule k CDS is of constant times larger than the minimal CDS. Hansen *et al* considered the situation that the size of the square (say L_N^2) grows linearly with N. The expected size of a CDS derived by restricted pruning rule -k is in an order linear to L_N^2 and lower bounded by L_N^2/π.

Along the same line of thought, we provide an alternative analysis on the size of a CDS derived by restricted pruning rule-k in this paper. The same assumptions that (i) $L_N \to \infty$ and (ii) $N \to \infty$ have also been made. Suppose the nodes are uniformly randomly generated in a square of $L_N \times L_N$. We would like to investigate how the size of such a CDS derived by restricted pruning rule-k changes with the node density, and when it reaches the lower bound as derived in [5].

The contribution of this paper are three folds.

(1) For a network of N nodes that are uniformly randomly generated in a square of size $L_N \times L_N$, the node degree distribution follows a Poisson distribution when $L_N, N \to \infty$.

(2) To obtain the probability unmarking of a marked node that is of degree d node, a procedure based on the idea of random sampling is proposed. The change of the probability or unmarking with respect to node degree is revealed.

(3) The expected size of a CDS that is derived by the restricted pruning rule is thus analyzed. It is found that the size of a CDS is almost a decreasing function with respect to the node density. The size of the CDSs reaches its lower bound when the node density is greater than or equal to 30.

[1] In this paper, the terms restricted rule-k and restricted pruning rule-k are used interchangeably.

The rest of the paper will be organized as follow. In the next section, the algorithm for marking and pruning will be presented. Specially, we restrict our attention to the latest version of the restricted pruning rule-k proposed by Dai & Wu in [2]. The idea of analysis is outlined in Section 3. The main results are followed in Section 4 to Section 7. In Section 4, the node degree distribution of a network of randomly deployed nodes is deduced and shown to be Poissonian. An empirical procedure to obtain the unmarked probability will be introduced in Section 5. In Section 6, the expected size will be analyzed, and a tighter lower bound is derived in Section 7. Section 8 gives a conclusion of the paper.

2 Pruning Rule

Consider a mobile ad hoc network of N nodes that are randomly uniformly depolyed within a two dimensional square of area $L \times L$. Because of the transmission power of a radio signal, two nodes can communicate with each other if their distance apart is less than an allowable transmission range, say r ($r \ll L$). In other words, node x and node y are neighbors to each other if

$$|\text{Location}(x) - \text{Location}(y)| < r.$$

Once a node has been deployed, (i) it generates a uniformly random ID for itself and broadcasts to other nodes nearby (if any) about its ID. Then, (ii) it waits and listens to the signals from nearby nodes about their IDs and the IDs of their neighbors. (iii) As long as the IDs have been received, it updates the list of the IDs of its neighbors and broadcasts this neighbor information to its neighbors. The listen-update-broadcast cycle is then repeated for a few more times until there are no more updates on the neighbor list. The resultant network graph is denoted by V.

When a complete list of neighbor information has been obtained, each node carries out the following algorithm to determine whether it is a gateway node (i.e. a node in a connected dominating set) for message routing. Let $id(x)$ be the ID and $\mathcal{N}(x)$ be the set of neighbor nodes of a node located at x. The marker of x is denoted by $\mathcal{M}(x)$.

Wu-Li Marking process [11]: A node located at x sets its marker to *True*, i.e. $\mathcal{M}(x) = T$, if there exists two unconnected neighbor nodes.

Dai-Wu Restricted Pruning Rule k [2]: A marked node unmarks itself if its neighbor nodes can be covered by a set of connected

neighbor nodes whose IDs are larger than node x. That is to say, $\mathcal{M}(x) = F$, if there exists $x_1, x_2, \cdots, x_k \in \mathcal{N}(x)$ such that

(i) $\mathcal{M}(x) = T$

(ii) $id(x) < id(x_j)$ for all $j = 1, 2, \cdots, k$.

(iii) $\mathcal{N}(x) \subset \mathcal{N}(x_1) \bigcup \mathcal{N}(x_2) \cdots \bigcup \mathcal{N}(x_k)$.

(iv) $x_1, x_2, \cdots, x_k$ form a connected graph.

To realize the above pruning rule efficiently, a marked node (say x) of degree d performs the following steps in practice. First, sort the IDs of its neighbor in ascending order. Then, select all nodes with IDs larger than $id(x)$ and form a graph. Check if the coverage of the graph can cover the unselected nodes (Step (iii)). If yes, chek if the graph is connected (Step (iv)). $\mathcal{M}(x)$ change to F if the selected nodes form a connected graph and can cover all the neighbors of x.

The beauty of this pruning rule is because of its distributed nature. Only direct neighbor information is needed. No global information is required. All calculations are done locally. Each node performs the marking and pruning processes locally. Besides, the computational complexity is small. For a node of degree d, the computational cost a node has to pay is just in an order of $\mathcal{O}(d^2)$.[2] Consider a graph of finite mean node degree (say μ) and variance (say σ^2), it can be shown by the Chebyshev Inequality that over $(1 - m^{-2})$ nodes are of degree in between $(\mu - m\sigma)$ and $(\mu + m\sigma)$.[3] In other words, over $(1 - m^{-2})$ nodes, their computational costs are just $\mathcal{O}(m^2\sigma^2)$.

3 Outline of analysis

Although the efficiency of Dai-Wu pruning algorithm has been demonstrated in [2], little analysis has been done on the size of a connected dominating set derived by the restricted pruning rule k algorithm.

Remember that the network graph before the marking process and pruning rule are performed is denoted by V and $|V| = N$. We further let V_{cds} be the graph of the virtual backbone induced by the connected dominating set formed after the pruning rule has been performed. We let $P(d)$ be the node degree distribution of the network graph before the marking process and pruning rule are performed. $P(\mathcal{M}(x) = F|deg(x) = d)$ is the probability that a node of degree d is unmarked after the pruning step.

[2] Theorem 4 in [2].

[3] In accordance with Chebyshev Inequality,

$$P(|d - \mu| \geq m\sigma) \leq \frac{1}{m^2}.$$

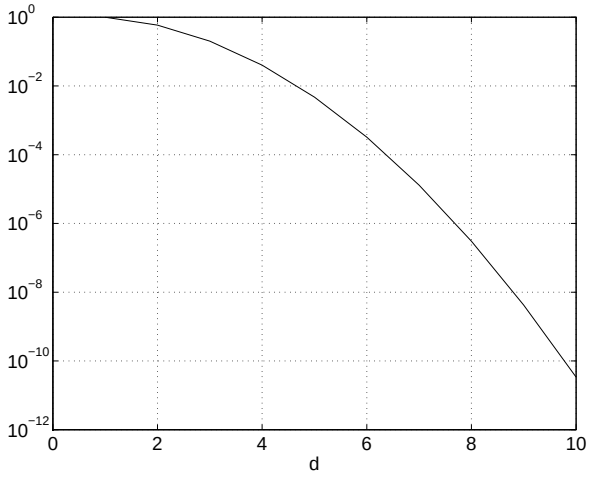

Figure 1: The probability that a node of degree d is not marked during the marking process.

During the marking process, a node will be marked if there exists two neighbor nodes of x that are not neighbor to each other. In other word, the probability that a node of degree d will be marked in the marking process is

$$(1 - \beta^{d(d-1)/2}),$$

where β is the probability that the distance of two random nodes within a unit circle is less than or equal to the radius.

By conducting a computer simulation that generates 10000 points uniformly randomly in a circle of radius r and then counts the percentage of pairs of nodes whose separation is less than r, it can find that β is equal to 0.5852. Then

$$P(\text{Node } x \text{ is marked}|deg(x) = d) = 0.995$$

for $d = 5$. Figure 1 shows the probability that a node of degree d is not marked during the marking process. Clearly, one can assume that this probability vanishes when $d > 6$.

Suppose a network graph is of Poisson node degree distribution with large mean node degree. As will be seen in the next section, the percentage of nodes of small node degree is very small. One can thus assume that all nodes are marked after the initial marking process has been performed. As $P(deg(x) = d)$ is homogenous for all $x \in V$, the expected size can simply be expressed as follows :

$$E[|V_{cds}|] = N \left(1 - \sum_d P(\mathcal{M}(d) = F|d)P(d)\right), \quad (1)$$

where the factor $P(\mathcal{M}(d) = F|d)$ is corresponding to the probability that a node of degree d is unmarked.

4 Node degree distribution

Suppose the nodes are randomly and uniformly distributed and let $\bar{n}$ be the average number of node within a unit disk. The average node degree λ will thus be $\hat{n} - 1$. The node degree distribution of V follows a Poisson distribution.

Theorem 1 *For a mobile ad hoc network V, in which the mobile nodes are randomly and uniformly distributed, the node degree distribution $P(d)$ is given by*

$$P(d) = \exp(-\lambda)\frac{\lambda^d}{d!}, \quad \lambda = \hat{n} - 1, \tag{2}$$

where $\hat{n}$ is the average node density.

(Proof) Let N be the total number of nodes of V, and the area of deployment is much larger than a unit disk. The number of nodes deployed within a unit disk will then follow a binomial distribution, the probability that exactly n nodes in a unit disk is

$$\frac{N!}{n!(N-n)!}\,\delta^n\,(1-\delta)^{(N-n)}.$$

where

$$\delta = \frac{\text{Area of a unit disk}}{\text{Deployment Area}}.$$

For large N, $\hat{n} = N\delta$ and the probability that exactly n nodes in a unit disk is

$$\exp(-\hat{n})\frac{\hat{n}^n}{n!}.$$

Therefore, the probability of a node having d node degree (i.e. the number of neighbor nodes) is given by a Poisson distribution with average node degree $\lambda = \hat{n} - 1$. **Q.E.D.**

Figure 2 shows two exemplar distributions in which $\hat{n}$ equals 20 and 10 respectively.

5 Unmarked probability

Recall that a marked node x unmarks itself if there exists $x_1, x_2, \cdots, x_k \in \mathcal{N}(x)$ such that

(i) $\mathcal{M}(x) = T$

(ii) $id(x) < id(x_j)$ for all $j = 1, 2, \cdots, k$.

(iii) $\mathcal{N}(x) \subset \mathcal{N}(x_1)\bigcup\mathcal{N}(x_2)\cdots\bigcup\mathcal{N}(x_k)$.

(iv) $x_1, x_2, \cdots, x_k$ form a connected graph.

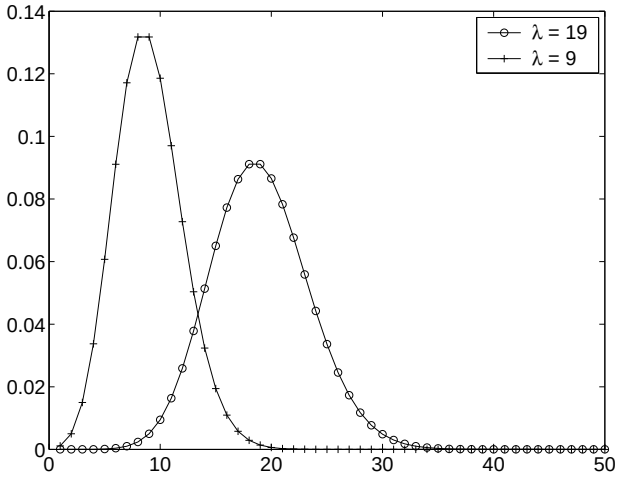

Figure 2: The node degree distributions of V for which the node densities are 20 ($\lambda = 19$) and 10 ($\lambda = 9$) respectively.

Consider the condition (i). As we have assumed that all the nodes are initially marked,

$$P(\mathcal{M}(x) = T) = 1 \quad \forall\, x \in V.$$

Thus, the node degree distribution of a connected dominating set after *marking process* can be treated as the same as before marking process.

Consider the condition (ii). For a node of degree d, it might have 1 neighbor node, 2 neighbor nodes, 3 neighbor nodes and so on, up to d neighbor nodes that have IDs larger than itself. Since all node IDs are uniformly randomly generated in a constant range, say $[0, 1]$, the probability that $id(x) < id(y)$ for all $y \in \mathcal{N}(x)$ is

$$\int_0^1 (1-u)du = \frac{1}{2}.$$

As a result, the probability that exactly k ($k \leq d$) neighbor nodes that have larger IDs is equal to

$$\left(\begin{array}{c} d \\ k \end{array}\right)\left(\frac{1}{2}\right)^d,$$

for all $k = 0, 1, 2, \cdots, d$.

The final question left behind is this : *If there are k neighbor nodes with larger IDs, will these nodes form a connected graph, and simultaneously will the rest of the other $d-k$ nodes be neighbors of these nodes.* Unfortunately, it is not an easy question. It all depends on the locations of these d neighbor nodes. Take a look at the illustrative examples shown in Figure 4 and Figure 3. In both cases, $k = 6$. Even though both sets of neighbor nodes can cover the whole circle, one is connected (Figure 4) and the other is disconnected (Figure 3).

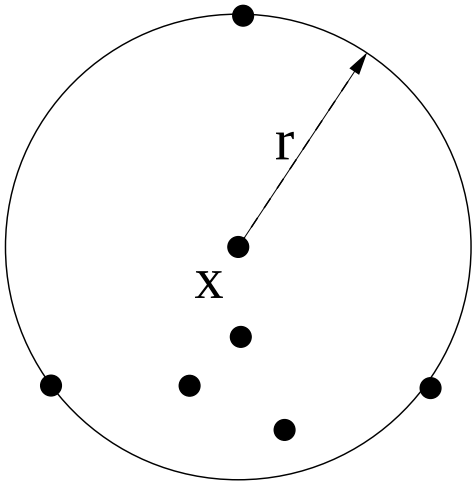

Figure 3: Disconnected neighbors that can cover the whole circle.

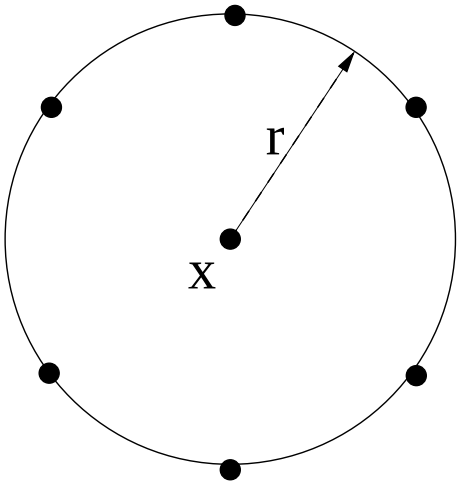

Figure 4: Connected neighbors that can cover the whole circle.

Let $\Omega(x)$ be the unit circle centered at location x. Let $X = (x_1, x_2, \cdots, x_k) \in \Omega(x)^k$ be an augmented random vector, in which $x_1, x_2, \cdots, x_k \in \Omega(x)$. The graph induced by X is denoted by G_X. Furthermore, we let $I(X)$ be an indicator function defined as follows :

$$I(X) = \begin{cases} 1 & \text{if } G_X \text{ is connected,} \\ 0 & \text{if } G_X \text{ is not connected.} \end{cases} \quad (3)$$

The coverage of X is denoted by $Cov(X)$, where

$$Cov(X) = \frac{\text{Area covered by } \bigcup_{j=1}^{k} (\Omega(x_j) \cap \Omega(x))}{\text{Area covered by } \Omega(x)}. \quad (4)$$

Therefore, the probability that $(d-k)$ random nodes in $\Omega(x)$ can be covered by the other k random nodes in $\Omega(x)$ will be given by

$$P(k,d) = \int_{X \in \Omega(x)^k} I(X) Cov(X)^{d-k} dX. \quad (5)$$

The probability that a node of degree d will be unmarked will thus be given by

$$P(\mathcal{M}(d) = F|d) = \sum_{k=1}^{d} P(k,d) \begin{pmatrix} d \\ k \end{pmatrix} \left(\frac{1}{2}\right)^d \quad (6)$$

and the expected size of CDS is given by

$$E[|V_{cds}|] = N\left(1 - \exp(\lambda) A(\lambda)\right), \quad (7)$$

where

$$A(\lambda) = \sum_{d} \sum_{k=1}^{d} P(k,d) \frac{(\lambda/2)^d}{k!(d-k)!}.$$

Unfortunately, there is no simple close form solution for the probability $P(k,d)$, Equation (5). We obtained the values empirically by the procedure depicted in Figure 5.

This procedure eventually generates a matrix PC of dimension 25×10000 and its kj^{th} element, i.e. PC_{kj} in Step 2.1.5, corresponds to the value $I(X)Cov(X)$ of the j^{th} set of random k nodes. The value $P(k,d)$ can then be obtained accordingly, i.e.

$$P(k,d) = \frac{1}{M} \sum_{j=1}^{M} PC_{kj}^{d-k} \quad (8)$$

for all $k \leq d$. The unmarked probability of a node of degree d can be obtained.

Figure 6 shows the unmarked probability $P(\mathcal{M}(d) = F|d)$ against node degree d. It is found that the minimum unmarked probability is attained at d equals to 5, which is equal to 0.3722. (It is due to the fact that there is a small chance for a 5-node induced graph to form a connected induced graph.) The unmarked probability reaches 0.9661 when $d = 25$. Further noted from the figure that the value of $P(\mathcal{M}(d) = F|d)$ increases as d increases and then approaches 1 when d is large.

Step 0: Initialize $OL, CN, PC \in R^{25\times 10000}$.

Step 1: Let Ω_0 be the unit disk centered at $(0,0)$ and then uniformly randomly generate $x_1, x_2, \cdots, x_{10000}$ inside Ω_0.

Step 2: For $k = 1, 2, \cdots, 25$,

Step 2.1: For $j = 1, 2, \cdots, 10000$

2.1.1: Uniformly randomly generate $y_1, y_2, \cdots, y_k$ inside Ω_0,

2.1.2: Set NI equals the number of x_is that are located inside $\bigcup_{\kappa=1}^{k} \Omega(y_\kappa) \bigcap \Omega_0$.

2.1.3: Set OL_{kj} equals $NI/10000$.

2.1.4: Set CN_{kj} equals 1 if $y_1, \cdots, y_k$ form a connected graph.

2.1.5: Set PC_{kj} equals $OL_{kj} \times CN_{kj}$.

Figure 5: Procedure for obtaining the probability $P(k, d)$, Equation (5).

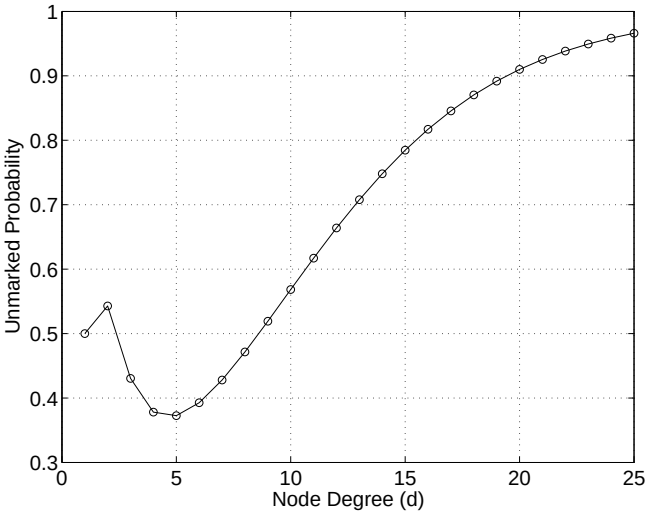

Figure 6: The probability that a node of degree d will be unmarked after the pruning process.

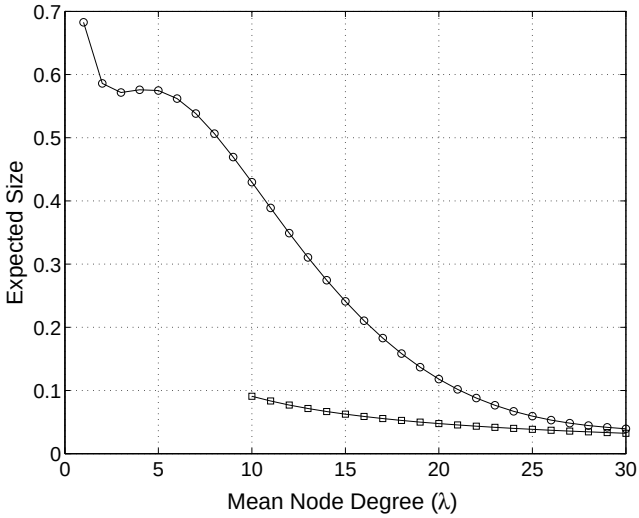

Figure 7: Expected size of a CDS derived by restricted pruning rule. The solid line with squares corresponds to the lower bound $(\lambda + 1)^{-1}$.

6 Expected size of CDS

In accordance with the formulae derived earlier for the average number of marked nodes (Equation (1)) and the theorem about the nature of node degree distribution (Theorem 1), the expected size of a CDs derived restricted pruning rule can be expressed as follows :

$$\frac{E[|V_{cds}|]}{N} = 1 - \exp(-\lambda)A(\lambda), \tag{9}$$

where

$$A(\lambda) = \sum_d P(\mathcal{M}(d) = F|d)\frac{\lambda^d}{d!}.$$

λ corresponds to the average node degree.

Then, the expected size of a CDS derived is evaluated by putting the values of $P(\mathcal{M}(d) = F|d)$ as shown in Figure 6 and different values of λ into the Equation (9). Figure 7 shows the expected size of CDS against λ. (For convenience, we simply let $P(\mathcal{M}(d) = F|d) = 0.9661$ for $d > 25$.) The solid line with squares corresponds to the lower bound $(\lambda+1)^{-1}$. (Please refer to Appendix A for the derivation of this lower bound, and a discussion on the lower bound of Equation (9).)

Clearly, the size is about 0.55% of the original network size when $\lambda = 6$. The factor matches the result obtained in [2] for the same λ and $N = 200$. (Please refer to Appendix B for the reason why the comparison is only made for $\lambda = 6$, not for other values of λ in their paper.) In accordance with Figure 7, one can also see that the size of a CDS drops as the λ increases. Eventually, it drops to Hansen *et al* Lower Bound when λ is close to 30.

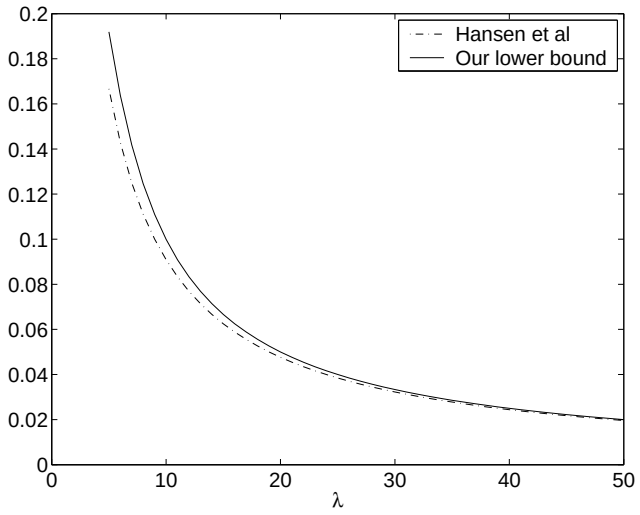

Figure 8: Comaprison between Hansen *et al* and our lower bounds.

7 A tighter lower bound

A tighter lower bound for Poissonian $P(d)$ can indeed be derived from the Equation (9). Consider a marked node of degree d. One condition that a marked node will be staying marked, after the pruning process, is when its ID is larger than all its neighbors. This probability is given by $(d+1)^{-1}$ for a marked node with d neighbors. Hence,

$$\frac{E[|V_{cds}|]}{N} \geq \exp(-\lambda) \sum_{d \geq 1} \frac{\lambda^d}{(d+1)!}.$$

Since

$$\sum_{d \geq 1} \frac{\lambda^d}{(d+1)!} = \frac{\exp(\lambda) - 1 - \lambda}{\lambda},$$

$$E[|V_{cds}|] \geq \left\{ \frac{1}{\lambda} - \frac{\lambda+1}{\lambda} \exp(-\lambda) \right\} N.$$

It is larger than $(\lambda+1)^{-1}N$ for all $\lambda \geq 3$. Figure 8 compares the difference between Hansen *et al* lower bound and ours lower bound. It is clear that there is no significant difference when λ is large.

8 Conclusion

In this paper, we have provided some analytical results on the size of a CDS derived by the restricted pruning rule-k algorithm. For a network of N nodes that are uniformly randomly generated in a square of size $L_N \times L_N$, we have shown that the node degree distribution follows a Poisson distribution when $L_N, N \to \infty$. To argue that the node degree distribution of the network does not change much after the marking process has been performed, we has discussed with an aid the probability of a node being marked in the marking process and shown that such a probability tends to vanish when the node density is high.

After that, we have derived an equation to evaluate the expected size of a CDS, in terms of the network node degree distribution and the unmarked probabilities. As there is no close form solution for the connected probability and the coverage of a graph induced by random nodes within a circle, a computer simulated procedure based on the idea of random sampling has been developed to obtain such probabilities. The probability that a node of degree d will be unmarked is obtained and hence the expected size of a CDS can be obtained. Finally, the expected size of a CDS derived by the restricted pruning rule-k is analyzed with respect to different node densities.

It is found that the size is almost a decreasing function with respect to the node density. The size reaches its lower bound when the node density is larger or equal to 30. That is to say, the CDS derived by the restricted pruning rule-k algorithm in a high node density situation is a minimal CDS. The results are consistent with the existing results previously obtained in [2] and [5]. More important, our results have filled in the gap, $6 \leq \lambda \leq 30$, that has not been investigated before. A tighter lower bound on the expected size has also been deduced.

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A Hansen *et al* Lower Bound

Instead of running extensive computer simulations, Hansen *et al* in [5] have presented a theoretical analysis on the size of a CDS derived by the restricted pruning rule. In one of their theorems[4], they show that

$$\text{Size of a CDS} \leq \frac{l_N^2}{\pi}$$

for $l_N \to \infty$. Here l_N is the length of the square where the mobile nodes are deployed.

For an ad hoc network consisting of N nodes, l_N^2/π is equal to the total number of nodes over the node density. As node density is equal to the average node degree plus 1, i.e.

$$\hat{n} = \lambda + 1,$$

[4]Theorem 5 in [5].

the lower bound of the expected size of a CDS derived by restricted pruning rule is depended on the average node degree of the Poissonian node degree distribution :

$$E[|V_{cds}|] \geq \frac{N}{\lambda + 1}. \tag{10}$$

It is a lower bound independent of the graphical structure of the network.

B Dai & Wu Result [2]

In our analysis, we assume the network is of Poisson node degree distribution. For a network of N nodes deployed in a square of size $L \times L$, and each node has transmission range r, it happens when $r \ll L$ and

$$\frac{r}{L} = \sqrt{\frac{\lambda + 1}{\pi N}}.$$

This condition is equivalent to $\lambda \ll N$ for L is finite. Therefore, only when λ is small, the node degree distribution is close to a Poisson distribution.

The simulated results in [2] for the expected size of a CDS at $\lambda = 6$ is consistent with our result obtained in this paper. On the contrary, the node degree distribution of a large λ network could hardly follow a Poisson distribution. Comparison between their results and the result presented here could not be made.