Optimizing Roadside Advertisement Dissemination in Vehicular Cyber-Physical Systems

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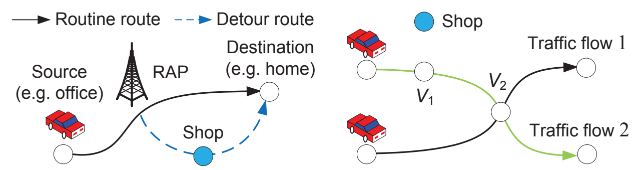
***Abstract*—In this paper, we address a promising application in the Vehicular Cyber-Physical Systems (VCPS) called *roadside advertisement dissemination*. Its application involves three elements: the drivers in the vehicles, Roadside Access Points (RAPs), and shopkeepers. The shopkeeper wants to attract as many customers as possible, through using RAPs to disseminate advertisements to the passing vehicles. Upon receiving an advertisement, the driver may detour towards the shop, depending on the detour distance. Given a fixed number of RAPs and the traffic distribution, our goal is to optimize the RAP placement for the shopkeeper to maximally attract potential customers. This application is a non-trivial extension of traditional coverage problems, the difference being that we use RAPs to cover the traffic flows. RAP placement algorithms may pose complex trade-offs. If we place RAPs at locations that can provide small detour distances to attract more customers, these locations may not necessarily be located in heavy traffic regions. While heavy traffic regions cover more flows, they can cause large detour distances, making shopping less attractive to customers. To balance this tradeoff, novel RAP placement algorithms are proposed. Since real-world traffic distributions exhibit unique patterns, here we further consider the Manhattan grid scenario and then propose improved solutions. Real trace-driven experiments validate the competitive performance of the proposed algorithms.**

***Keywords*-Vehicular cyber-physical systems, advertisement dissemination, placement, coverage problem.**

I. Introduction

Vehicular Cyber-Physical Systems (VCPS) refer to a new generation of vehicular systems; VCPS integrate computational and physical capabilities that can interact with humans through many new modalities [1]. While traditional vehicular systems were generally considered to be a common component of the physical world, VCPS actively interact with humans through communications which yield a very tight coordination between cyber and physical resources [2]. Hence, it is critical for VCPS to consider the perceptions and reactions of humans (i.e., the drivers in the vehicles). The effectiveness and efficiency of VCPS depend on how humans could benefit from such a system [3].

In this paper, we address a novel and promising application in VCPS called *roadside advertisement dissemination* [4]. Its application involves three basic elements: the drivers in the vehicles (the human factor), Roadside Access Points (RAPs), and shopkeepers. The shopkeeper wants to attract as many customers as possible, by using RAPs to disseminate electronic advertisements to passing vehicles. The drivers may decide to go shopping or not upon receiving advertisements, depending on the detour distance. An example is shown in Fig. 1(a), where commuters drive home after work. During their trip home, they receive an advertisement from an RAP, and then, decide to detour to the shop. We observe that the driver may not shop if the detour distance to the shop is too large. This is because the desire to shop does not outweigh the cost of the journey. If the shop is on the driver's way home, he or she may stop by due to convenience. We focus on the scenario with one shop; however, our model can be easily extended to scenarios with multiple shops.



(a) The detour model. (b) The RAP placement.

Figure 1. The scenario of the roadside advertisement dissemination.

Fig. 1(b) shows some traffic flows on the streets. Each traffic flow represents a group of commuters traveling home from work. Given a fixed number of RAPs and the traffic distribution that can be obtained from the previous records, we focus on optimizing the RAP placement for the shopkeeper, as to maximally attract potential customers. Our problem is a non-trivial extension of traditional coverage problems, the difference being that we use RAPs to cover the traffic flows. A tradeoff exists between the traffic density and the detour probability for the RAP placement problem. Let us consider the placement of only one RAP in Fig. 1(b). If we place the RAP at V_1 near the shop, then this RAP can only cover traffic flow 2, while traffic flow 1 is not covered. On the other hand, if we place the RAP at V_2, although both traffic flows 1 and 2 are covered, the nearby drivers are not likely to go shopping, due to a large detour distance. Our problem becomes more challenging when placing multiple RAPs. Let us consider the placement of two RAPs in Fig. 1(b). Suppose these two RAPs are placed at V_1 and V_2, respectively. Then, the RAP at V_2 is meaningless for the drivers in traffic flow 2, since the RAP at V_1 provides a smaller detour distance for those drivers. Redundant advertisements do not provide additional shopping incentives. If a driver decides not to go shopping despite a smaller detour distance, then they would not go shopping with a larger detour distance. Accordingly, the geographical distribution of RAPs should also be controlled.

Furthermore, the real-world traffic distributions exhibit certain patterns, which can be utilized for the RAP placement. For example, the streets in Manhattan are mapped as a grid, meaning that all the vehicles have only four possible moving directions. This means that the RAP placement is more controllable. Another interesting observation is that multiple shortest paths exist, connecting a pair of locations in the Manhattan grid streets. These properties pose several unique challenges regarding RAP placement optimization.

Our main contributions are summarized as follows:

- We address a novel and promising application called roadside advertisement dissemination, which follows the design principle of VCPS. The model is a non-trivial extension of traditional coverage problems.
- Two utility functions are used to model the driver's detour probability. Greedy solutions with ratios of $1-1/e$ and $1-1/\sqrt{e}$ to the optimal solutions are proposed for those two utility functions, respectively.
- Since the real-world traffic distributions exhibit certain patterns, we further study the RAP placement problem in the Manhattan grid street scenario, where we propose solutions with tightened bounds.
- Extensive experiments are conducted to evaluate the proposed solutions. The results are provided from different perspectives to provide insightful conclusions.

The remainder of this paper is organized as follows. Section II surveys the related work. In Section III, the general models are described. In Section IV, we discuss the Manhattan grid scenario. Section V includes the experiments. Finally, we conclude the paper in Section VI.

II. Related Work

Cyber-physical systems are engineered systems whose operations are monitored, coordinated, controlled and integrated by a computing and communication core [5]. VCPS are special types of cyper-physical systems designed for vehicles. While traditional protocols are imperceptible by humans, VCPS take humans' perceptions into account [6]. For example, the data dissemination mechanism in [7] considers the data to be location-dependent, the paradigms of which humans are not aware of. By contrast, Li et al. [3] considered a human-oriented service scheduling in VCPS, where a driver cannot receive multiple services in a short time. Wagh et al. [8] proposed that the data composition in VCPS should be flexible for drivers.

Currently, the advertisement dissemination is considered as a novel and promising application in VCPS [9], since advertisements belong to the practically useful data. While traditional studies focus on online advertisements [10–12], Li et al. [4] first considered the advertisement dissemination

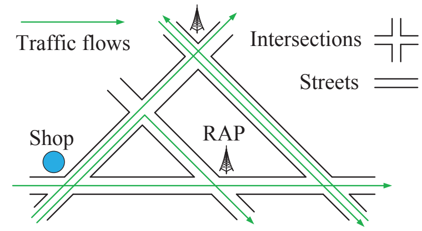


Figure 2. The scenario for the RAP placement problem.

in VCPS as a bandwidth allocation problem with pre-fixed locations of RAPs. We optimize the RAP placement for the advertisement dissemination. This application is also studied from different perspectives. Shen et al. [13] studied the message authentication problem for safety advertisements.

Our RAP placement problem is also related to the existing set cover problems [14], since RAPs are used to cover the passing vehicles. However, our problem cannot be solved by existing techniques, since the detour distance is considered. The location chosen for a driver to receive the advertisement is critical for his/her detour decision. Although the set cover problem has been well-solved with some greedy approximation algorithms, our problem brings more unique challenges.

III. General Models and Solutions

A. Model and Problem Formulation

As shown in Fig. 2, our advertisement dissemination scenario is based on a directed graph $G = (V, E)$, where V is a set of nodes (i.e., street intersections), and $E \subseteq V^2$ is a set of directed edges (i.e., streets). Some traffic flows exist on the streets. We assume that all cars start from and stop at intersections. Let $T_{i,j}$ denote the traffic flow from intersections i to j (e.g., vehicles that return home from the office). The traveling path for $T_{i,j}$ is unique and is known a priori (a shortest path in general). Let $|T_{i,j}|$ denotes the number of drivers in $T_{i,j}$. Then, T is the set of traffic flows that are targeted for the advertisement dissemination, where $|T|$ is the number of traffic flows in T. Traffic flows with insufficient potential customers are not counted in T.

We start with the scenario with only one shop. To attract customers, the shopkeeper places a fixed number, k, of RAPs at street intersections for the advertisement dissemination. A placed RAP would send electronic advertisements to all passing vehicles. Depending on the detour distance, drivers make a decision of whether or not to shop. For each traffic flow, $T_{i,j} \in T$, its detour distance is denoted by $d_{i,j}$ and can be calculated as follows. (i) Suppose $T_{i,j}$ only goes through one RAP, as shown in Fig. 3. When the driver receives the advertisement, the shortest path distance from the current location to the shop is d', from the shop to the destination j is d'', and from the current location to the destination j is d'''. Then, we have $d_{i,j} = d' + d'' - d'''$. (ii) If the traffic flow $T_{i,j}$ goes through multiple RAPs, then the corresponding detour distance should be the minimum detour distance among all these RAPs. This is because redundant advertisements do not

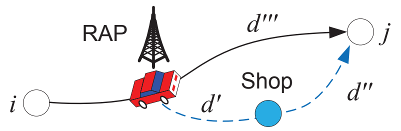


Figure 3. The calculation of the detour distance.

provide additional shopping incentives. If a driver decides not to go shopping despite a smaller detour distance, then they would not go shopping with a larger detour distance. Our model can be extended to scenarios with multiple shops. For those cases, the result depends on the shop that provides the smallest detour distance among all the shops.

The roadside advertisement dissemination is an application in VCPS, where the human perceptions should be considered upon designing the system. Therefore, we use a utility function, $f(d_{i,j})$, to describe the detour probability of the driver. The driver may not shop if the detour distance to the shop is too large. This is because the desire to shop does not outweigh the cost of the journey. Hence, $f(d_{i,j})$ should be non-increasing with respect to $d_{i,j}$. In this paper, two kinds of utility functions are considered, namely threshold utility function and decreasing utility function. The former one means that the detour probability is a certain constant, if $d_{i,j}$ is no greater than a threshold, D. It is shown as:

$$f(d_{i,j}) = \begin{cases} \alpha(T_{i,j}) & \text{if } d_{i,j} \leq D \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

In Eq. 1, $\alpha(T_{i,j})$ shows the advertisement attractiveness for the drivers in the traffic flow $T_{i,j}$. This is known a priori through previous statistics. Then, the decreasing utility function means that $f(d_{i,j})$ is strictly decreasing from $\alpha(T_{i,j})$ to 0 with respect to $d_{i,j}$. An example could be:

$$f(d_{i,j}) = \begin{cases} \alpha(T_{i,j}) \cdot (1 - d/D) & \text{if } d_{i,j} \leq D \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

For each traffic flow $T_{i,j}$, an expectation of $f(d_{i,j}) \cdot |T_{i,j}|$ drivers would detour to the shop as potential customers.

In this paper, we study the RAP placement problem. Given a fixed number of RAPs and the traffic distribution, our goal is to optimize the RAP placement for the shopkeeper to maximally attract potential customers. RAP placement algorithms may pose complex trade-offs. If we place RAPs at locations that can provide small detour distances to attract more customers, these locations may not necessarily be located in heavy traffic regions, i.e., $f(d_{i,j})$ is large but $|T_{i,j}|$ is small. While heavy traffic regions cover more flows, they can cause large detour distances, making shopping less attractive to customers, i.e., $|T_{i,j}|$ is large but $f(d_{i,j})$ is small. Another challenge lies on the geographical density of the placed RAPs. Redundant advertisements do not provide additional shopping incentives for humans. Hence, it is unnecessary to place too many RAPs within a small area. In the next two subsections, we will propose bounded solutions for the two kinds of utility functions, respectively.

Algorithm 1 A greedy solution

Input: The directed graph G; the set of traffic flows T; The number of RAPs to place (i.e., k);
Output: The RAP placement;

```
Mark all the traffic flows as uncovered;
for i = 1 to k do
    Among all the intersections, find the one that attracts
    maximum drivers from the uncovered traffic flows;
    Place an RAP at that intersection, and then mark the
    corresponding traffic flows as covered;
return the RAP placement;
```

B. *RAP Placement with Threshold Utility Function*

In this subsection, we discuss the RAP placement problem, using the threshold utility function in Eq. 1. The detour probability is fixed, if the detour distance is no greater than a threshold. Then, we have the following two definitions:

***Definition** 1:* An intersection *includes* a traffic flow, if (i) this traffic flow goes through this intersection, and (ii) the detour distance for this traffic flow at this intersection is no greater than the threshold D in Eq. 1.

***Definition** 2:* An RAP *covers* a traffic flow, if this RAP is placed at an intersection that includes this traffic flow.

If a traffic flow is included by multiple intersections, then placing an RAP at any of these intersections can cover this traffic flow. If a traffic flow is covered by one RAP, then using more RAPs to cover this traffic flow does not attract more drivers to the shop as potential customers (redundant advertisements do not bring additional shopping incentives).

It turns out that our RAP placement problem with the threshold utility function is essentially a weighted maximum coverage problem [15], which is an NP-hard problem, as follows. First, there are some sets defined over a domain of elements associated with weights. The goal of the weighted maximum coverage problem is to select k sets, such that the total weight of elements within the selected sets is maximized. In our problem, the elements correspond to the traffic flows, and the sets correspond to the intersections. The weight of a traffic flow is its number of expected drivers that detour to the shop, if this traffic flow is covered. The selection of a set corresponds to the placement of an RAP.

It is well-known that the weighted maximum coverage problem has a greedy algorithm that can achieve a ratio of $1-1/e$ to the optimal solution [16]. We can use that greedy algorithm to solve the RAP placement problem, as shown in Algorithm 1. We iteratively place an RAP at an intersection that attracts the maximum number of drivers from uncovered traffic flows. The geographical density of the RAPs can be controlled by Algorithm 1, since the covered traffic flows are no longer considered for the subsequent RAP placement. An RAP is not likely to be placed near an existing RAP, since the nearby traffic flows have been covered. The algorithm

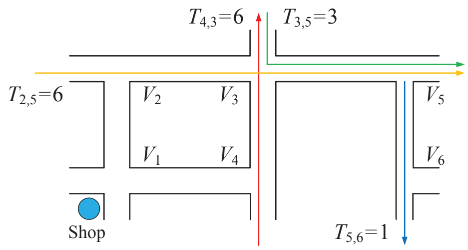


Figure 4. An example of the RAP placement ($k = 2$ and $D = 6$).

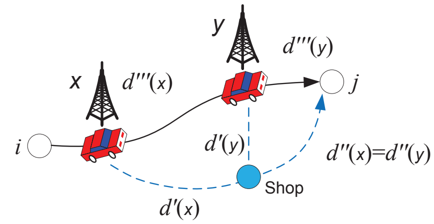


Figure 5. An illustration for the proof of Theorem 1.

complexity is $O(|V|^3 + k|V||T|)$, where k is the number of RAPs, $|V|$ is the total number of intersections, and $|T|$ is the total number of traffic flows. $O(|V|^3)$ results from the calculation of detour distances, since we need to calculate the shortest paths between all pairs of nodes. $O(k|V||T|)$ comes from the greedy approach. Each greedy step takes $O(|V||T|)$ to examine all the intersections. Examining each intersection takes $O(|T|)$ to check all the traffic flows.

For a better explanation, an example is shown in Fig. 4, where we have two RAPs ($k = 2$) to place. The distance between neighboring intersections is 1. The threshold D is 6. The shop is located at V_1. The $\alpha(T_{i,j})$ in Eq. 1 is set to be 1 for all the traffic flows. This example involves four traffic flows, which are initialized as uncovered. In the first step, V_3 is picked to place an RAP, since it can attract the maximum amount of drivers from uncovered traffic flows ($|T_{2,5}|$+$|T_{3,5}|$+$|T_{4,3}|$=15). Then, these three traffic flows are marked as covered. The remaining uncovered traffic is $T_{5,6}$. Therefore, the second RAP is placed at V_5 to cover $T_{5,6}$. The algorithm terminates for this example, since all the traffic flows are covered. Note that V_6 does not include $T_{5,6}$, since its detour distance is 8 (the path changes from V_5V_6 to $V_5V_6V_5V_3V_2V_1V_2V_3V_5V_6$). Since the detour distance is larger than the threshold D, the driver would not detour to the shop, upon receiving the advertisement at V_6.

The RAP placement problem with the threshold utility function is solvable. This is because the tradeoff between the traffic density and the detour probability is weakened by the threshold utility function, where the detour probability is discrete (either zero or a fixed constant). We can eliminate the intersections with zero detour probabilities to simplify the RAP placement problem. In contrast, the RAP placement problem with the decreasing utility function is a non-trivial extension of traditional coverage problems.

C. RAP Placement with Decreasing Utility Function

Here, we discuss the RAP placement problem with the decreasing utility function. For a better explanation, let us revisit the example in Fig. 4 with the decreasing utility function in Eq. 2. Suppose the two RAPs are still placed at V_3 and V_5, which is the optimal placement with the threshold utility function. For the decreasing utility function, the detour probability for $T_{2,5}$ and $T_{4,3}$ at V_3 is $1 \times (1 - \frac{4}{6}) = \frac{1}{3}$ with a detour distance of 4 (the path for $T_{2,5}$ changes from $V_2V_3V_5$ to $V_2V_3V_4V_1V_4V_3V_5$). $T_{3,5}$ is covered by two RAPs. However, the drivers in $T_{3,5}$ would detour at V_3 rather than V_5, since the detour distance at V_3 is smaller than that at V_5. Therefore, the detour probability for $T_{3,5}$ is $1 \times (1 - \frac{4}{6}) = \frac{1}{3}$ with a detour distance of 4. Meanwhile, the detour probability for $T_{5,6}$ is 0 with a detour distance of 6. Therefore, the total number of detoured drivers is $(6 + 6 + 3) \times \frac{1}{3} = 5$. However, a better strategy is to place these two RAPs at V_2 and V_4, respectively. Under this strategy, the detour probability for $T_{2,5}$ is $1 \times (1 - \frac{2}{6}) = \frac{2}{3}$ with a detour distance of 2 (the path changes from $V_2V_3V_5$ to $V_2V_1V_2V_3V_5$). The detour probability for $T_{4,3}$ is also $\frac{2}{3}$, and the total number of detoured drivers is $(6+6) \times \frac{2}{3} = 8$. This placement strategy attracts more drivers, since it takes the detour distance into consideration. Although V_3 and V_5 have more passing traffic flows, the corresponding detour distances are larger. We should further consider the detour distance for optimizing the RAP placement.

Instead of placing RAPs at the intersections that can cover the maximum uncovered traffic, we can place RAPs at the intersections that can attract maximum drivers under the existing placement. If we go back to the example in Fig. 4, then the first RAP would be placed at V_3, since it attracts the maximum amount of drivers ($(6+6+3) \times (1 - \frac{4}{6}) = 5$) at the first step. Then, an RAP is placed at V_2 at the second step, since $[6 \times (1 - \frac{2}{6})] - [6 \times (1 - \frac{4}{6})] = 2$ more drivers can be attracted. This is because the drivers in $T_{2,5}$ can have a smaller detour distance at V_2. However, this solution only attracts $2 + 5 = 7$ drivers to the shop, while placing these two RAPs at V_2 and V_4 is still a better strategy.

The key insight behind the above phenomenon involves the overlaps between RAPs. When the second RAP is placed at V_2, the first RAP at V_3 becomes useless for the traffic flow $T_{2,5}$. This is because the second RAP at V_2 provides a smaller detour distance than that at V_3. Let us consider a driver in $T_{2,5}$. If this driver decides not to go shopping at V_2, he/she would make the same decision at V_3 due to the larger detour distance. Although the two RAPs at V_2 and V_3 both cover $T_{2,5}$, the one at V_2 provides more powerful coverage due to the smaller detour distance. In terms of $T_{2,5}$, those two RAPs have overlapping coverage, which is the key challenge for the RAP placement. If a traffic flow goes through multiple RAPs, then the corresponding detour probability depends on the RAP that provides the minimum detour distance. Further analysis shows the following result:

Algorithm 2 A composite greedy solution

Input: The directed graph G; the set of traffic flows T; The number of RAPs to place (i.e., k);

Output: The RAP placement;

Mark all the traffic flows as uncovered;
for $i = 1$ to k **do**
  Candidate intersection i: Among all the intersections, find the one that attracts a maximum amount of drivers from the uncovered traffic flows;
  Candidate intersection ii: Among all the intersections, find the one that attracts a maximum amount of additional drivers from the covered traffic flows, through providing smaller detour distances;
  Compare intersections i and ii, place an RAP at the one that can attract more drivers to the shop;
  Mark the corresponding traffic flows as covered;
return the RAP placement;

***Theorem** 1:* For a specified traffic flow $T_{i,j}$ that goes from i to j, the first RAP on its path always provides a smaller detour distance than all the other RAPs on the path.

Proof: As shown in Fig. 5, let us select two arbitrary RAPs (say x and y) that cover the traffic flow $T_{i,j}$. Then, the difference between the detour distances of x and y is:

$$\begin{aligned}&[d'(x){+}d''(x){+}d'''(x)] - [d'(y){+}d''(y){+}d'''(y)]\\ &= d'(x) - \{d'(y){+}[d'''(x){-}d'''(y)]\} < 0\end{aligned} \tag{3}$$

In Eq. 3, $\{d'(y){+}[d'''(x){-}d'''(y)]\}$ is the distance from x to the shop via y. Since $d'(x)$ is the shortest path from x to the shop, it is smaller than $\{d'(y){+}[d'''(x){-}d'''(y)]\}$. Hence, the detour distance at x is smaller than that at y. Since x and y are arbitrarily selected, Theorem 1 is true. ■

Theorem 1 shows that the first RAP on the path of a traffic flow provides the smallest detour distance for this traffic flow. If a driver decides not to go shopping at the first RAP, he/she would make the same decision at later RAPs due to the larger detour distance. Therefore, for a specified traffic flow, the first RAP overlaps with the subsequent RAPs for the decreasing utility function. The subsequent RAPs are useless for this traffic flow, since none of the drivers would detour at the subsequent RAPs. The insight is that the first RAP provides the highest traveling flexibility for the drivers.

To maximally attract potential customers for the shopkeeper, the RAP placement involves two critical factors. (i) The first factor is to find an intersection, the RAP placement at which a maximum amount of drivers can be attracted from the uncovered traffic flows. This factor is similar to that for the threshold utility function. (ii) The second factor is to find an intersection, the RAP placement at which maximum additional drivers can be attracted from the covered traffic flows. This factor represents the overlaps among RAPs, where we want to find intersections that can provide smaller detour distances for the covered traffic flows. If we consider only one of the above factors, then the resulting greedy algorithm cannot guarantee an approximation bound. Therefore, Algorithm 2 is proposed based on a composite greedy objective using two factors. At each greedy step, Algorithm 2 picks out the better one from two candidate intersections that are corresponding to the above two factors: (i) attract drivers from uncovered traffic flows, and (ii) attract additional drivers from covered traffic flows by providing smaller detour distances, i.e., overlaps among RAPs. The performance of Algorithm 2 is guaranteed as follows:

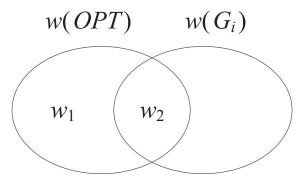


Figure 6. An illustration for the proof of Theorem 2.

***Theorem** 2:* Algorithm 2 achieves a ratio of $1-1/\sqrt{e}$ to the optimal solution, in terms of maximizing the number of attracted drivers who detour to the shop.

Proof: Let OPT denote the optimal RAP placement. G_i is the RAP placement in Algorithm 2, after the first i steps. G_i should include i RAPs and G_k is the final RAP placement obtained by Algorithm 2. $G_{i+1}\backslash G_i$ is the RAP placed at the $(i{+}1)^{th}$ step. Let $w(\cdot)$ be a function that denotes the number of attracted drivers for the corresponding RAP placement. Then, let us focus on the difference between OPT and G_i. As shown in Fig. 6, OPT can attract more drivers than G_i, for the following two reasons. (i) OPT may cover some traffic flows that are uncovered by G_i. The number of attracted drivers for this part is denoted as w_1. (ii) OPT may provide smaller detour distances for some traffic flows in G_i. The number of additional attracted drivers for this part is denoted as w_2. Then, we have

$$w(OPT) - w(G_i) \leq w_1 + w_2 \tag{4}$$

The "$\leq$" in Eq. 4 results from the fact that G_i may cover some traffic flows that are uncovered by OPT, or G_i may also provide smaller detour distances for some traffic flows in OPT. Let us focus on the $(i+1)^{th}$ step in Algorithm 2. Since the candidate intersection i in Algorithm 2 attracts maximum drivers from the uncovered traffic flows, we have

$$\frac{w_1}{k} \leq w(G_{i+1}\backslash G_i) = w(G_{i+1}) - w(G_i) \tag{5}$$

OPT includes k RAPs, the average of which should cover no greater traffic flows than candidate intersection i in Algorithm 2 at the $(i+1)^{th}$ step, due to its greedy nature. Meanwhile, $w(G_{i+1}\backslash G_i)$ should be no less than the number of attracted drivers by the candidate intersection i, since Algorithm 2 picks the better one among the two candidate

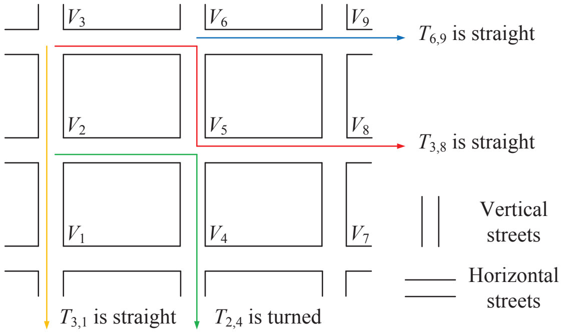


Figure 7. An illustration of the Manhattan grid streets.

intersections. Similarly, for the overlaps, we have

$$\frac{w_2}{k} \leq w(G_{i+1} \backslash G_i) = w(G_{i+1}) - w(G_i) \tag{6}$$

This is because the candidate intersection ii attracts a maximum amount of additional drivers from the covered traffic flows. Combining Eqs. 4, 5, and 6, we have

$$\frac{w(OPT) - w(G_i)}{2k} \leq w(G_{i+1}) - w(G_i) \tag{7}$$

Since $w(OPT) \geq w(G_i)$ and $w(OPT) \geq w(G_{i+1})$ by the definition of the optimal solution, Eq. 7 can be rewritten as

$$w(OPT) - w(G_i) \geq \frac{2k}{2k-1}[w(OPT) - w(G_{i+1})] \tag{8}$$

Considering $w(G_0) = 0$ means that no RAP has been placed at the beginning, the recursion in Eq. 8 leads to

$$\begin{aligned} w(OPT) &\geq \left[\frac{2k}{2k-1}\right]^k [w(OPT) - w(G_k)] \\ &\geq \sqrt{e} \cdot [w(OPT) - w(G_k)] \end{aligned} \tag{9}$$

This is because $e = lim_{k \to \infty}(1+\frac{1}{k})^k$ by its definition. Then, Eq. 9 can be rewritten as

$$w(G_k) \geq (1 - \frac{1}{\sqrt{e}}) \cdot w(OPT) \tag{10}$$

Since G_k is the result of Algorithm 2, Eq. 10 means that it achieves a ratio of $1 - 1/\sqrt{e}$ to the optimal solution. ■

Algorithm 2 guarantees the performance ratio through considering both covering uncovered traffic flows and providing covered traffic flows with smaller detour distances. It reduces to Algorithm 1, if we use the threshold utility function. In terms of the time complexity, Algorithm 2 also takes $O(|V|^3 + k|V||T|)$. $O(|V|^3)$ results from the calculation of detour distances, through finding out the shortest paths between all pairs of nodes. $O(k|V||T|)$ comes from the greedy approach, which has k steps. At each greedy step, searching for the candidate intersection i takes a time complexity of $O(|V||T|)$, since we need to examine all the intersections for all the traffic flows. Searching for the candidate intersection ii also takes $O(|V||T|)$, since the first RAP on the path provides the smallest detour distance.

IV. RAP Placement for Manhattan Grid

Considering that the real-world traffic distributions have some patterns, in this section, we look into a special RAP placement under the Manhattan grid scenario.

A. Properties of Manhattan Grid

The Manhattan grid streets plan is a type of city plan in which streets run at right angles to each other. In this city plan, vehicles can only move in four given directions, as shown in Fig. 7. We classify the streets into vertical streets and horizontal streets, based on their orientations. Multiple shortest paths between pairs of intersections may exist in the Manhattan grid streets. For example, in Fig. 7, the shortest path from V_1 to V_6 could be $V_1V_2V_3V_6$, $V_1V_2V_5V_6$, and $V_1V_4V_5V_6$. We relax the constraint used in the previous section, where the traveling path for a traffic flow is unique and is known a priori. In this section, the traveling path for a traffic flow is not pre-fixed. Let us consider a driver in $T_{1,6}$ that travels from V_1 to V_6. His/Her traveling path would be randomly chosen from among the three shortest paths. At this time, what could happen if an RAP is placed at V_3? This driver would definitely choose $V_1V_2V_3V_6$ as his/her traveling path, since it is one of the shortest paths with a free additional advertisement. We consider a traffic flow, $T_{i,j}$, to travel along one of the shortest paths from i to j; if an RAP is placed in one of the shortest paths, then the traffic flows would choose that path to obtain a free additional advertisement. Locations of placed RAPs are assumed to be known by all the drivers (they are published on the internet).

Considering the above property, we reformulate the RAP placement problem under the Manhattan grid scenario, as follows. The shop is located within a square region. D is large enough such that vehicles would detour to to the shop once receiving an advertisement. All the traffic flows travel through their shortest paths. For a specified traffic flow, if an RAP is placed in one of its shortest paths, this traffic flow would travel through that path to obtain a free advertisement.

B. Manhattan RAP Placement

Let us start with the Manhattan RAP placement problem under the threshold utility function. This problem remains NP-hard, since it still reduces to the maximum coverage problem. An intersection can still include multiple traffic flows, while a traffic flow can be included by multiple intersections. We start with the following definition:

***Definition** 3:* A traffic flow is *turned* if it has exactly one turn within the grid. Otherwise, it is *straight*.

For example, in Fig. 7, the traffic flows of $T_{3,1}$ and $T_{6,9}$ are straight, while $T_{2,4}$ is turned. Note that $T_{3,8}$ is straight, since it has two turns at V_5 and V_6, respectively. All the traffic flows have at most two turns within the scenario, otherwise, the corresponding traveling path is not a shortest path. Then, we observe that a turned traffic flow has multiple

Algorithm 3 A two-stage solution

Input: The square region; the set of traffic flows T; The number of RAPs to place (i.e., k);

Output: The RAP placement;

1: **if** $k \leq 5$ **then**
2:     **return** the optimal solution by exhaustive search;
3: **for** each corner of the square region **do**
4:     Place an RAP at that corner;
5: **for** $i = 1$ to $k - 4$ **do**
6:     Among all the intersections, place an RAP at the one that attracts maximum marginal number of drivers;
7: **return** the RAP placement;

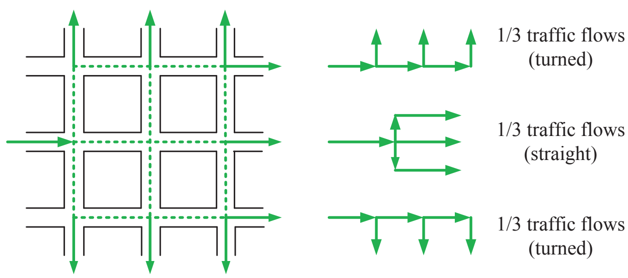


Figure 8. Traffic flow distribution.

shortest traveling paths. Therefore, the locations of RAPs may have some impacts on its actual traveling path.

Following the above intuition, a two-stage RAP placement algorithm is proposed as Algorithm 3. The RAPs are placed for turned and straight traffic flows, respectively. Algorithm 3 assumes that the traffic flows are uniformly distributed. We start with the case in which all the traffic flows go through the scenario. For this case, turned and straight traffic flows have fractions of $\frac{2}{3}$ and $\frac{1}{3}$ with respect to the total traffic flows, respectively. The reason is illustrated in Fig. 8. We have two observations. (i) Four RAPs at the corners of the grid can cover all the turned traffic flows. This is because corresponding drivers could go to the corner for a free advertisement without extra traveling distances. (ii) The RAP placement for the straight traffic flows can be obtained through the same idea of Algorithm 1. This placement keeps a ratio of $1 - \frac{1}{e}$ to the optimal solution. Cases are similar, when traffic flows may start from or stop at an intersection within the scenario. Consequently, we have:

Theorem 3: On expectation, Algorithm 3 has a ratio of $1 - \frac{1}{3e} - \frac{4}{3k}$ to the optimal solution, under the Manhattan grid scenario with the threshold utility function.

Proof: Our proof includes two parts. The first part is for the turned traffic flows and the second part is for the straight traffic flows. In the first part, we claim that four RAPs at the corners of the grid are enough to cover all the turned traffic flows that go through the scenario (lines 3 and 4 in Algorithm 3). Let us consider a traffic flow that enters the grid via the west boundary of the grid, and exits the grid via the south boundary of the grid. Such a traffic flow could be $T_{2,4}$ in Fig. 7. The shortest paths for this traffic flow only include two kinds of orientations: going Eastward or going Southward at an intersection. If this traffic flow goes Southward to the end and then goes Eastward, it would result in a shortest path that goes through the southwest corner of the grid. For example, such a shortest path for $T_{2,4}$ in Fig. 7 is $V_2V_1V_4$ that goes through the corner V_1. Since $V_2V_1V_4$ is a shortest path for $T_{2,4}$ with a free advertisement, drivers in $T_{2,4}$ would choose $V_2V_1V_4$ as their traveling paths. By enumerating all the possibilities, our claim is true. On expectation, four RAPs at the corners of the grid can cover $\frac{2}{3}$ of the total traffic flows. The above analysis is also valid, when traffic flows may start from or stop at an intersection within the scenario (i.e., not go through the scenario).

In the second part, we focus on the straight traffic flows. The greedy placement in lines 5 and 6 of Algorithm 3 has a ratio of $1 - \frac{1}{e}$ to the optimal solution using $k - 4$ RAPs. This is similar to Algorithm 1. Furthermore, it has a ratio of $\frac{k-4}{k}(1 - \frac{1}{e})$ to the optimal solution using k RAPs. This is because four RAPs are placed at the corner of the scenario for turned traffic flows. Note that, straight traffic flows have a fraction of $\frac{1}{3}$ with respect to all the traffic flows on expectation. Considering both turned traffic flows and straight traffic flows, the total fraction of traffic flows that are covered by Algorithm 3 is:

$$\frac{2}{3} + \frac{1}{3} \cdot \frac{k-4}{k} \cdot (1 - \frac{1}{e}) \geq 1 - \frac{1}{3e} - \frac{4}{3k} \tag{11}$$

Eq. 11 completes the proof of Theorem 3. ■

Algorithm 4 A modified two-stage solution

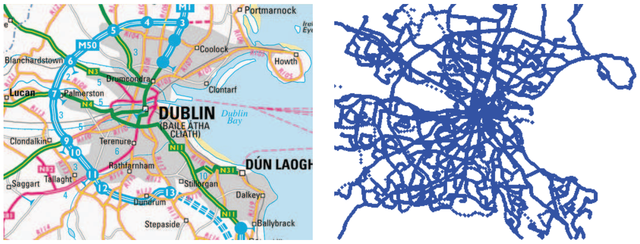
Input: The square region; the set of traffic flows T; The number of RAPs to place (i.e., k);

Output: The RAP placement;

1: Same as Algorithm 3, except the change of lines 3 and 4: For each corner of the square region, an RAP is placed in the middle of that corner and the center of the square;

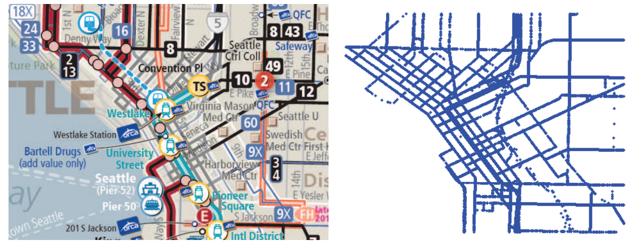
When k becomes larger, $1 - \frac{1}{3e} - \frac{4}{3k}$ becomes larger, meaning that Algorithm 3 has a better performance. $1 - \frac{1}{3e} - \frac{4}{3k}$ is larger than $1 - \frac{1}{e}$ when $k > 5$. Then, let us discuss the Manhattan RAP placement problem under the decreasing utility function. Similarly, we place RAPs for turned and straight traffic flows, respectively. However, the overlaps among RAPs bring some performance degradations. Algorithm 4 is proposed as an extension of Algorithm 3. It has a prerequisite in which the decreasing utility function must be the one in Eq. 2. The performance of Algorithm 4 is also guaranteed (the proof is omitted due to space limitation):

Theorem 4: On expectation, Algorithm 4 has a ratio of $\frac{1}{2} - \frac{1}{6e} - \frac{2}{3k}$ to the optimal solution, under Manhattan grid scenario with the decreasing utility function.



(a) The Dublin map. (b) The bus traces.

Figure 9. The map and bus traces for Dublin's central area.



(a) The Seattle map. (b) The bus traces.

Figure 10. The map and bus traces for Seattle's central area.

V. Evaluations

A. Real Trace-driven Datasets and Basic Settings

In this section, we conduct experiments based on two real traces, i.e., Dublin bus trace [17] and Seattle bus trace [18]. The city plan of Dublin is not grid-based, and thus the Dublin bus trace is used to test our Algorithms 1 and 2 for the general scenario in Section III. The city plan of Seattle is partially grid-based, and thus the Seattle bus trace is used to test our algorithms for both the general scenario in Section III and the Manhattan grid scenario in Section IV.

For the Dublin bus trace, we focus on the part within Dublin's central area, which is a $80,000 \times 80,000$ square foot area, as shown in Fig. 9. The Dublin bus trace includes bus ID, longitude, latitude, and vehicle journey ID. The vehicle journey is a given run on a journey pattern, which corresponds to our concept of the traffic flow. Buses with the same vehicle journey ID have similar routing paths in terms of longitude and latitude. To obtain the number of attracted customers, we assume that each bus in Dublin carries 100 people (who are potential customers) per day on average.

For the Seattle bus trace, we focus on the part within Seattle's central area, which is a $10^4 \times 10^4$ square foot area, as shown in Fig. 10. The Seattle bus trace includes bus ID, x-coordinate, y-coordinate, and route ID. Each route is regarded as a traffic flow. Buses with the same route ID have similar routing paths in terms of x and y coordinates. To obtain the number of attracted customers, we assume that each bus in Seattle carries 200 people per day on average.

According to the amount of passing traffic flows, all the street intersections in both traces are classified into the city's center, city, or suburb. This is used to observe the impact of the shop location. Our experiments use three utility functions. The first one is the threshold utility function in Eq. 1. The second one is the decreasing utility function i in Eq. 2, which decays linearly. The third one is the decreasing utility function ii, as defined in the following:

$$f(d_{i,j}) = \begin{cases} \alpha(T_{i,j}) \cdot (1 - \sqrt{d/D}) & \text{if } d_{i,j} \leq D \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Under the same detour distance, d, and the same threshold, D, the detour probability of the threshold utility function is the largest, that of the decreasing utility function i is in the middle, and that of the decreasing utility function ii is the smallest. In these three utility functions, $\alpha(T_{i,j})$ is set to be 0.001 for all the traffic flows. This means that a person receiving advertisements has a probability of 0.001 to go shopping [4], if the shop is on the way.

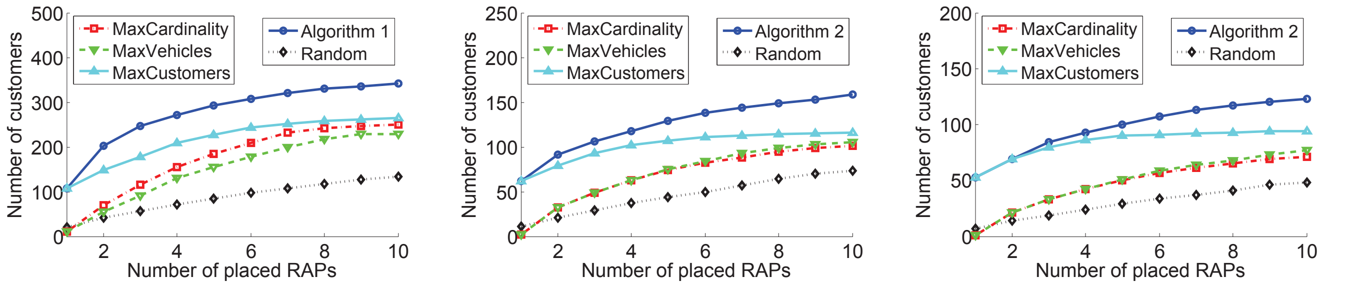
B. Comparison Algorithms and Metrics

In our experiments, four baseline algorithms (MaxCardinality, MaxVehicles, MaxCustomers, and Random) are used for comparisons, as in the following. (i) MaxCardinality ranks the intersections by the number of passing traffic flows, and then places the RAPs at the top-k intersections. (ii) MaxVehicles ranks the intersections by the number of passing buses, and then places the RAPs at the top-k intersections. (iii) MaxCustomers ranks the intersections by the number of attracted customers if an RAP is placed. Then, MaxCustomers also places RAPs at the top-k intersections. (iv) Random places RAPs uniform-randomly at the intersections within the $D \times D$ square region centered at the shop.

Our experiments focus on the relationship between the number of placed RAPs and the number of attracted customers, under different settings (utility functions, threshold D, and shop locations). Street intersections are classified into city's center, city, or suburb, depending on the amount of passing traffic flows. In the following experiments, if we say that the shop is located in the city, it means that intersections with city tags are randomly selected as the shop locations. All the results are averaged over 1,000 times.

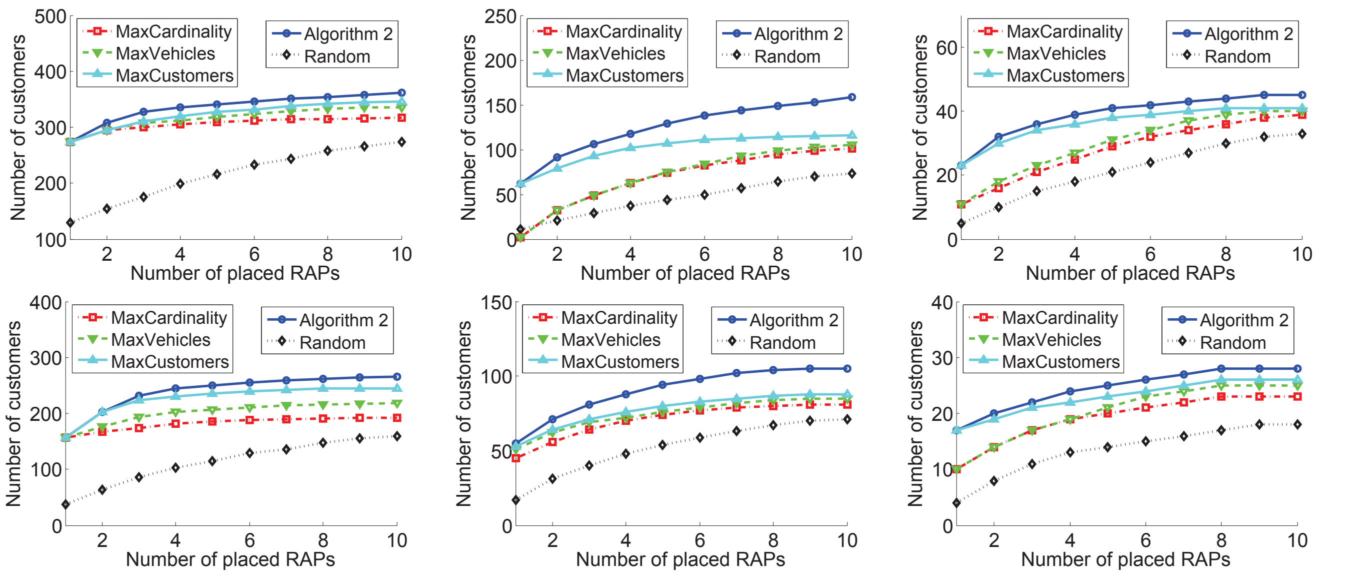
C. Evaluation Results in the Dublin Bus Trace

The evaluation results in the Dublin bus trace under the general scenario are shown in Figs. 11 and 12. Fig. 11 focuses on the impact of the utility function, where the shop is located in the city with the threshold $D = 20,000$ feet. It can be seen that all algorithms attract more customers under the threshold utility function than the decreasing utility functions i and ii. This is because the detour probability of the threshold utility function is the largest among the three utility functions, under the same d and D. In Fig. 11(a) with the threshold utility function, the performance gap between the Algorithm 1 and the other algorithms is significant (around 30% performance gain for $k = 10$). This is because Algorithm 1 controls the geographical density of the RAPs. In Fig. 11(b) with the decreasing utility function i, Algorithm 2 also outperforms the others. This is because it



(a) Threshold utility function. (b) Decreasing utility function i. (c) Decreasing utility function ii.

Figure 11. The experimental results for the Dublin bus traces with different utility functions. The shop is located in the city where $D = 20,000$ feet.



(a) The shop is located in the city's center. (b) The shop is located in the city. (c) The shop is located in the suburb.

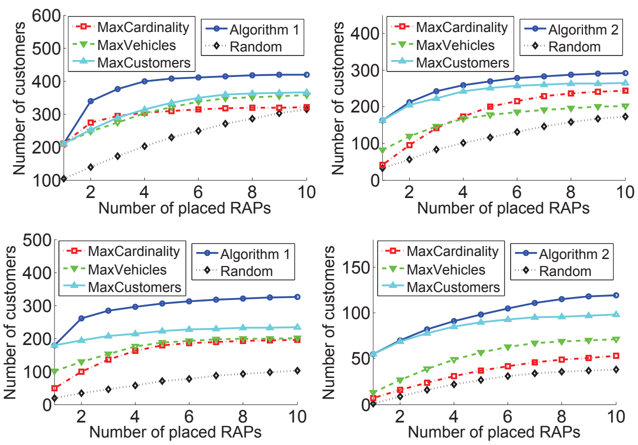
Figure 12. The experimental results for the Dublin bus traces with different shop locations. The decreasing utility function i is used with a different D.

has considered the overlaps among RAPs. However, in Fig. 11(c) with the decreasing utility function ii, Algorithm 2 has a smaller performance gain. This is because the decreasing utility function ii decays very fast with respect to the detour distance, and we have to place RAPs around the shop.

Fig. 12 shows the impact of the shop location and the threshold D, under the decreasing utility function i. Figs. 12(a), 12(b), and 12(c) show the results for different shop locations. For each subfigure, the top and bottom parts show the results with D=$20,000$ and D=$10,000$ feet, respectively. The performance gain of Algorithm 2 is relatively small, when the shop is located in the city's center or suburb. If the shop is located in the city's center, randomly placing RAPs around the shop can already cover most traffic flows with small detour distances. On the other hand, if the shop is located in the suburb, none of the placement strategies can cover too many traffic flows. The threshold D is also critical. A larger D means that the drivers are more likely to detour to the shop, and thus, the shop can attract more customers. When the shop is located in the city's center, a large D does not bring too many additional customers, since most traffic flows are already near to the shop. When the shop is located in the suburbs, a large D still does not bring too many additional customers, since the detour distances are large. However, when the shop is located in the city, as shown in Fig. 12(b), a large D brings many more customers, since more traffic flows are covered with small detour distances.

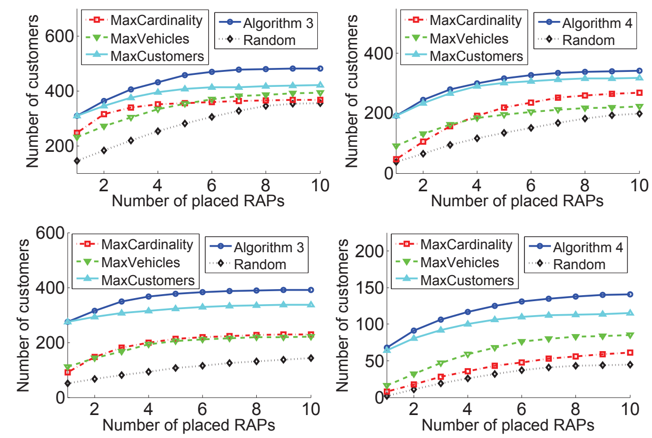
D. Evaluation Results in the Seattle Bus Trace

The evaluation results in the Seattle bus trace under the general scenario and the Manhattan grid scenario are shown in Figs. 13 and 14, respectively. The shop is located in the city. We focus on the impacts of different utility functions with a different threshold D. The threshold utility function is used in Figs. 13(a) and 14(a), while the decreasing utility function is used in Figs. 13(b) and 14(b). For each subfigure, the top and bottom parts show the results with D=$2,500$ and D=$1,000$ feet, respectively. In Fig. 13, it can be seen that all algorithms attract more customers under the threshold utility function than the decreasing utility functions i, since the former one brings higher detour probabilities. D is also critical, especially when the shop is located in the city. The



(a) Threshold utility function. (b) Decreasing utility function i.

Figure 13. The experimental results for the Seattle bus traces under the general scenario in Section III. The shop is located in the city. Different utility functions with different threshold D are used.



(a) Threshold utility function. (b) Decreasing utility function i.

Figure 14. The experimental results for the Seattle bus traces under the Manhattan grid scenario in Section IV. The shop is located in the city. Different utility functions with different threshold D are used.

number of attracted customers with $D{=}2,500$ feet is 30% more than that with $D{=}1,000$ feet. Fig. 14 shows the results in the Seattle bus trace under the Manhattan grid scenario. Compared with the results under the general scenario in Fig. 13, more customers are attracted under the Manhattan grid scenario. This is because the traveling paths of all the traffic flows are pre-fixed under the general scenario, the assumption of which is relaxed here. A larger threshold D also brings more customers to the shop.

VI. Conclusions

In this paper, we address a novel roadside advertisement dissemination problem that involves elements: the drivers, RAPs, and shopkeepers. The shopkeeper uses RAPs to disseminate advertisements to the drivers, as to attract customers. Upon receiving an advertisement, the driver may detour to the shop, depending on the detour distance. Our goal is to optimize the RAP placement for the shopkeeper to maximally attract potential customers. Bounded RAP placement algorithms are proposed. Real trace-driven experiments validate the competitive performance of our algorithms.

VII. Acknowledgments

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