A Budgeted Framework to Model a Multi-round Competitive Influence Maximization Problem

Nadia Niknami, Jie Wu

Computer and Information Science Department
Temple University

Outline

- ❑ Models of influence
  - Linear Threshold
  - Independent Cascade
- ❑ Influence maximization problem Algorithm
- ❑ Competitive Influence maximization problem
- ❑ Our Approach
- ❑ Experiments
  - Data and setting
  - Results

What is Social Influence?

- Social network plays a fundamental role as a medium for the spread of **INFLUENCE** among its members
  - Opinions, ideas, information, innovation…

- **Nodes**: Social actors (individuals or organizations)
- **Links**: Social relations

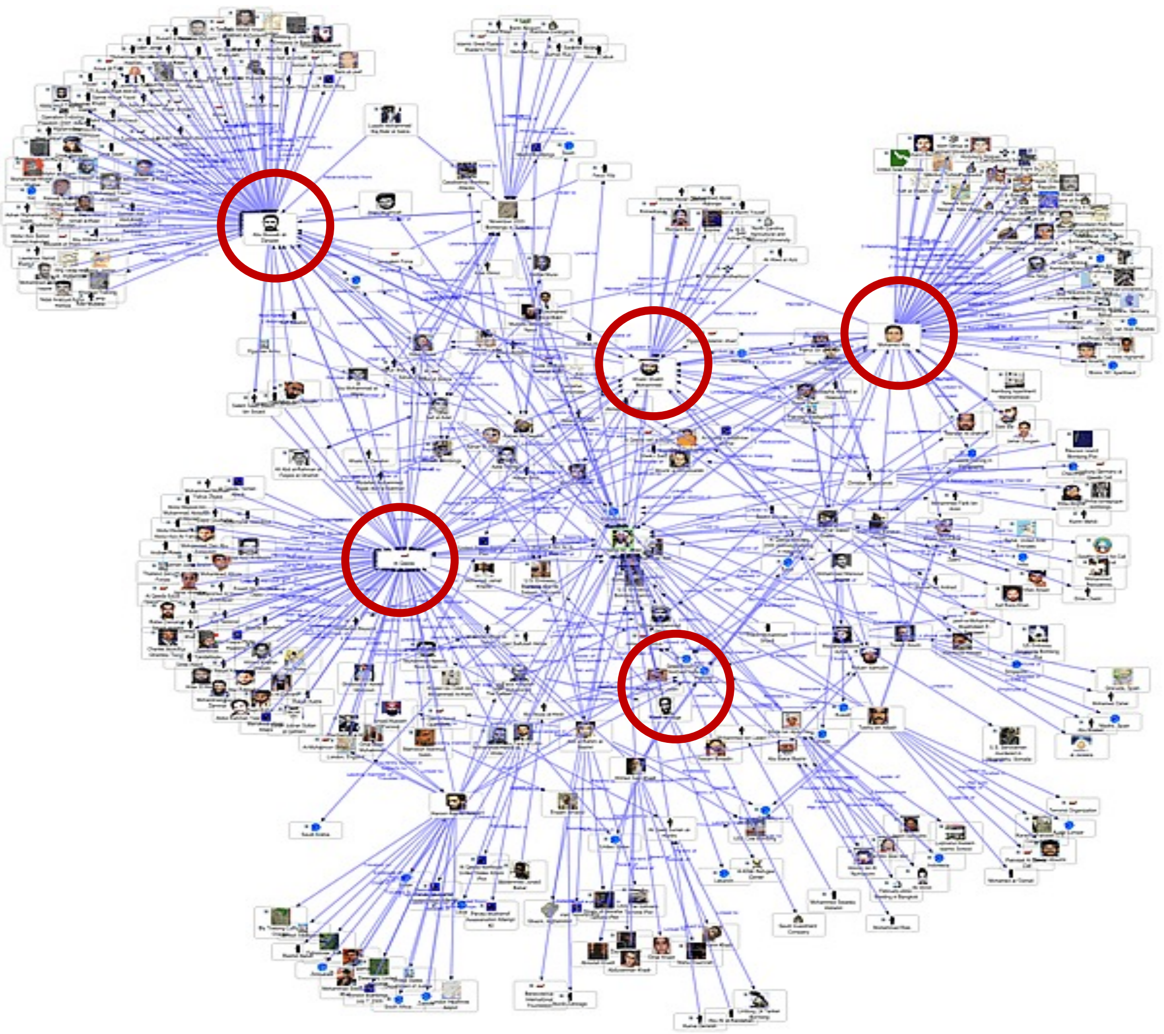
- Social influence occurs when one's opinions, emotions, or behaviors are affected by others, intentionally or unintentionally.

- Influential persons often have many friends.

Influential Persons

- Number of friends: Node degree
- Famous persons
- Betweenness centrality

- Direct Marketing takes the "word-of-mouth" effects to significantly increase profits



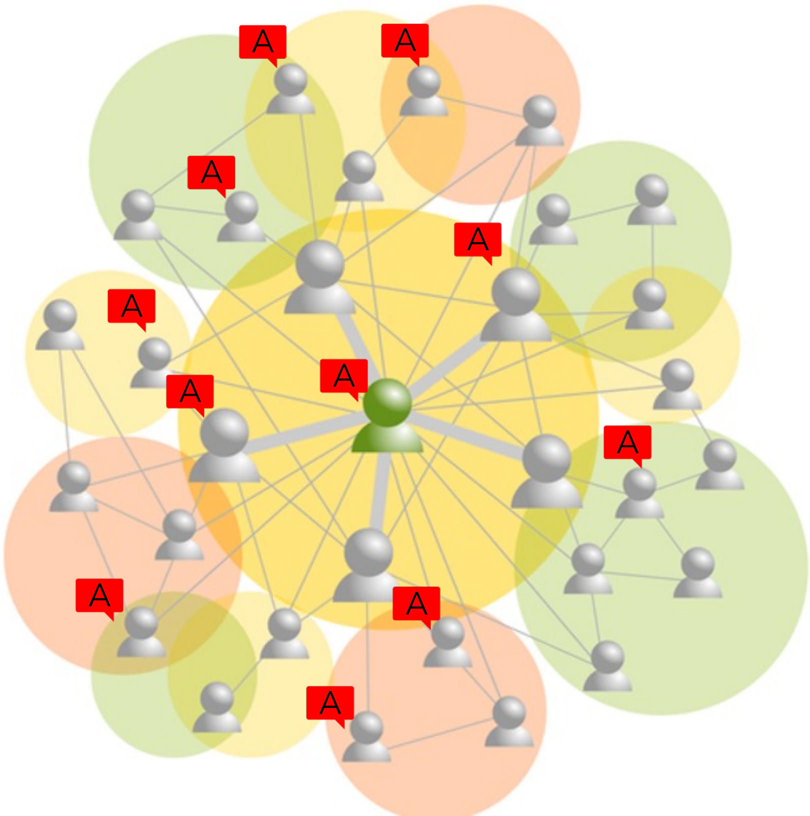
Influence Maximization Problem

- Influence spread of node set S: $\sigma(S)$
  - expected number of active nodes at the end of diffusion process, if set S is the initial active set.
- Problem Definition (by Kempe et al., 2003):

  (Influence Maximization). Given a directed and edge-weighted social graph $G = (V, E, p)$, a diffusion model m, and an integer $k \leq |V|$, find a set $S \subseteq V$, $|S| = k$, such that the expected influence spread $\sigma(S)$ is maximum.

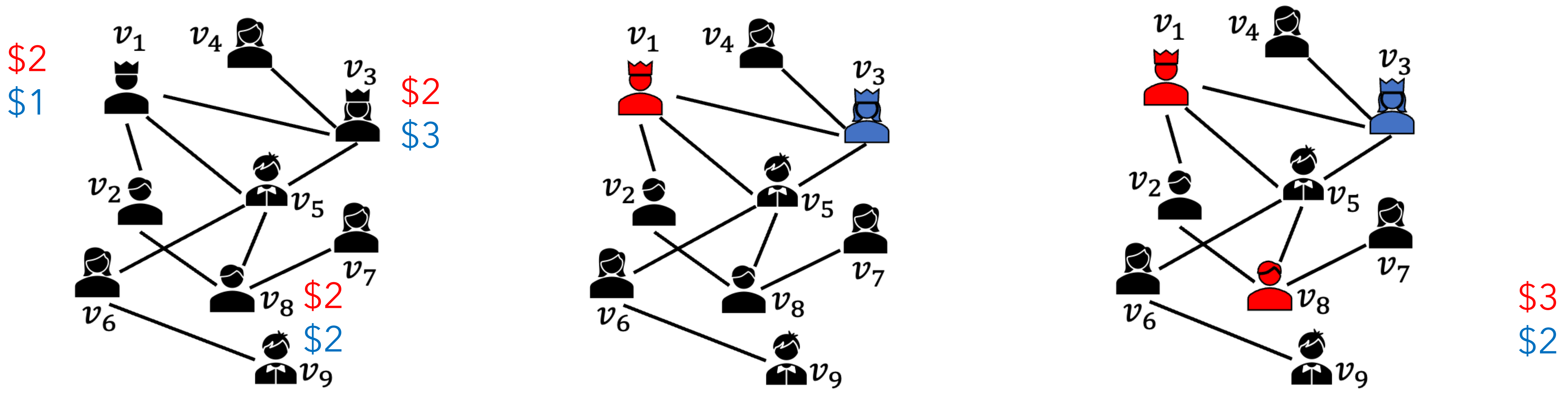
Influence Maximization Problem

- Diffusion Model
  - Linear Threshold (LT)
    - A node has random threshold θ
    - A node becomes active when at least θ fraction of its neighbors are active
  - Independent Cascade (IC)
    - When node v becomes active, it has a single chance of activating each currently inactive neighbor w.
    - The activation attempt succeeds with probability p_{vw}



Competitive Influence Maximization

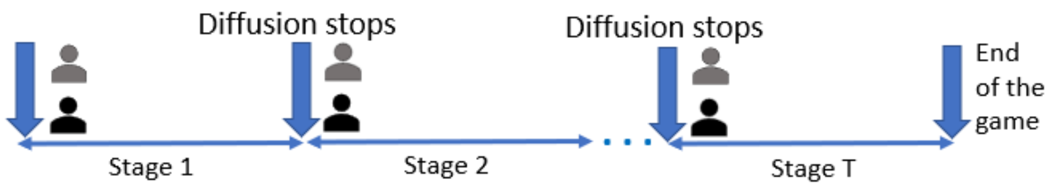
- First, parties identify the most influential nodes of the network.
- Then they compete over only these influential nodes by the amount of budget they allocate to each node.



Multi-round Diffusion

- In a multi-stage CIM problem, competitors need to select seed nodes simultaneously in each of the sequence stages.

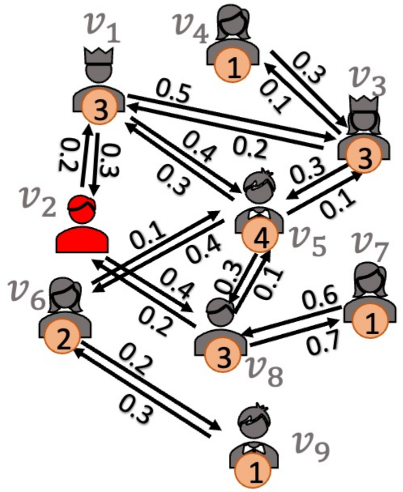
- 1) Nod-Node influence competition
- 2) Link-Link influence competition
- 3) Node-Link influence competition

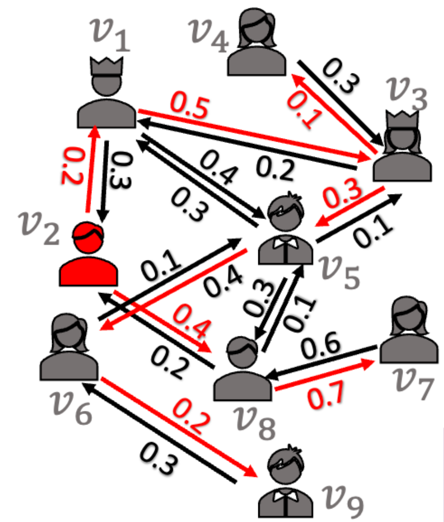


A Budgeted Framework to Model a Multi-round Competitive Influence Maximization Problem

- We integrate seed selection and budget allocation into the RL model.
- Selecting Seed Nodes and Propagation Model
- Most Reliable Influence Path (MRIP)
    - Fixed weight
    - Dynamic weight
- Reinforcement Learning

MRIP





$$w'(u) = \sum_{\forall v \in V} R(v) \times w(u),$$

$$w'(v_1) = 3 * R(v_3) = 3$$
$$w'(v_2) = 2 * \left(R(v_1) + R(v_8)\right) = 1.2$$
$$w'(v_3) = 3 * \left(R(v_4) + R(v_5)\right) = 1.2$$
$$w'(v_4) = 0$$
$$w'(v_5) = 4 * R(v_6) = 0.48$$
$$w'(v_6) = 2 * R(v_9) = 0.048$$
$$w'(v_7) = 0$$
$$w'(v_8) = 3 * R(v_7) = 0.56$$
$$w'(v_9) = 0$$

Algorithm 2 Finding seed set by MRIP

$S \leftarrow \varnothing$
for all $u \in V$ **do**
  $w'(u) \leftarrow 0$
for $u \in V$ **do**
  Construct $\mathcal{T}_u$ via Alg. 3
  for each leaf v in reverse $\mathcal{T}_u$ **do**
    $z \leftarrow parent(v)$
    while $v \neq u$ **do**
      Compute $w'(z) = w'(z) + R(v) \times w(z)$
      $v \leftarrow z$
      $z \leftarrow parent(v)$
new seed $\leftarrow \arg\max_{u \in V/\ S} w'(u)$
$S \leftarrow S \cup \{new\ seed\}$
$V_A \leftarrow$ Activated nodes by *new seed* node
Constructing $\mathcal{G}'$ with vertex set $V - V_A$
Recalculate $\mathcal{T}$ and w' in $\mathcal{G}'$

Algorithm 3 Computing $\mathcal{T}_u$

Require: $\mathcal{G}(V, E, P)$, source node u
$A = \{u\}$, $R(u) = 1$
while $A \neq V$ **do**
  Find node $v \in N(A)$ and $v \in V - A$ such that
  $R'(v) = \max_{(s,v): s \in A, v \in V-A} R(s) \times p(s, v)$
  $R(v) = R'(v)$
  $A = A \cup \{v\}$
  Set s as the parent of v in spanning tree $\mathcal{T}_u$
return $\mathcal{T}_u$

Most Reliable Influence Path (MRIP)

v_2

v_2, v_8

v_2, v_8, v_7

v_2, v_8, v_7, v_1

v_2, v_8, v_7, v_1, v_3

$v_2, v_8, v_7, v_1, v_3, v_5$

$v_2, v_8, v_7, v_1, v_3, v_5, v_6$

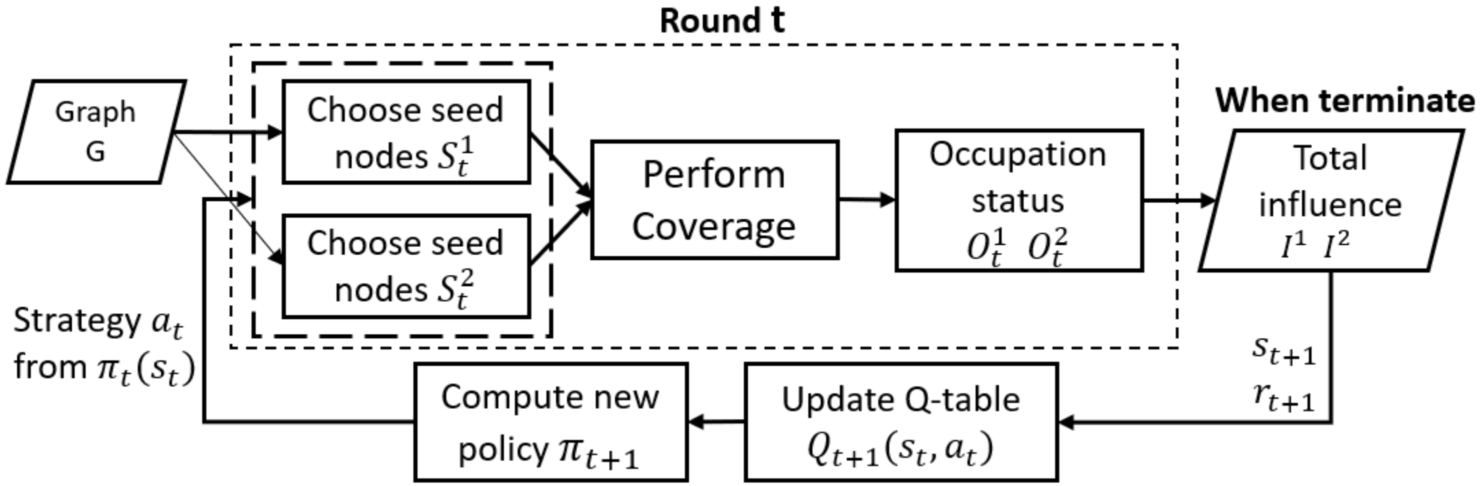
$v_2, v_8, v_7, v_1, v_3, v_5, v_6, v_4$

$v_2, v_8, v_7, v_1, v_3, v_5, v_6, v_4, v_9$

A	N(A)	R(s)* p(s,v)	R(v)
$\{v_2\}$	v_1	$1 * 0.2 = 0.2$ **1*0.4=0.4**	$R(v_8)$
$\{v_2, v_8\}$	v_1 v_5 v_7	$1 * 0.2 = 0.2$ $0.4 * 0.1 = 0.04$ **0.4*0.7=0.28**	$R(v_7)$
$\{v_2, v_8, v_7\}$	v_1 v_5	**1*0.2=0.2** $0.4 * 0.1 = 0.04$	$R(v_1)$
$\{v_2, v_8, v_7, v_1\}$	v_1 v_5	$0.4 * 0.1 = 0.04$ **0.2*0.5=1** $0.2 * 0.4 = 0.08$	$R(v_3)$
$\{v_2, v_8, v_7, v_1, v_3\}$	v_1 v_5	$0.4 * 0.1 = 0.04$ $0.2 * 0.4 = 0.08$ $1 * 0.1 = 0.1$ **1*0.3=0.3**	$R(v_5)$
$\{v_2, v_8, v_7, v_1, v_3, v_5\}$	v_4 v_6	$1 * 0.1 = 0.1$ **0.3*0.4=0.12**	$R(v_6)$
$\{v_2, v_8, v_7, v_1, v_3, v_5, v_6\}$	v_4 v_9	**1*0.1=0.1** $0.12 * 0.2 = 0.024$	$R(v_4)$
$\{v_2, v_8, v_7, v_1, v_3, v_5, v_6, v_4\}$	v_9	**0.12*0.2=0.024**	$R(v_9)$

Reinforcement Learning

- In RL, the agent keeps interacting with the environment to find the optimal policy π to maximize his expected accumulated rewards
- State
- Action



Reinforcement Learning

State

- 1) Number of inactive nodes
- 2) Summation of degrees of all inactive nodes
- 3) Maximum degree among all inactive nodes
- 4) Summation of the weight of the edges for which both vertices are inactive
- 5) Summation of the inactive out-edge weight for nodes which are the neighbors of player I
- 6) Maximum sum of the inactive out-edge weight of a node among all nodes
- 7) Ratio of budgets
- 8) Weight of nodes in case of reachability

Action:

- (1) Selecting a new seed node and
- (2) feeding a node in case of tie.

Experiment

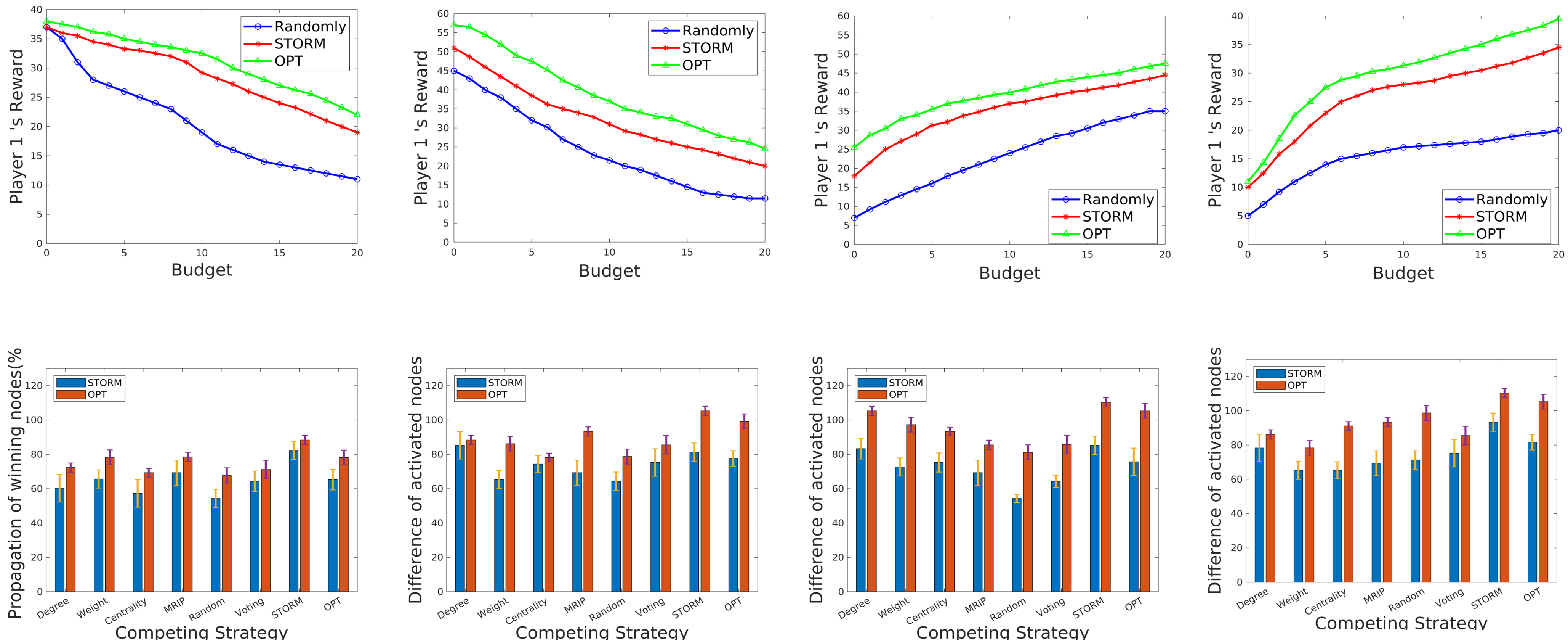
1)Evaluation on Budget Setting

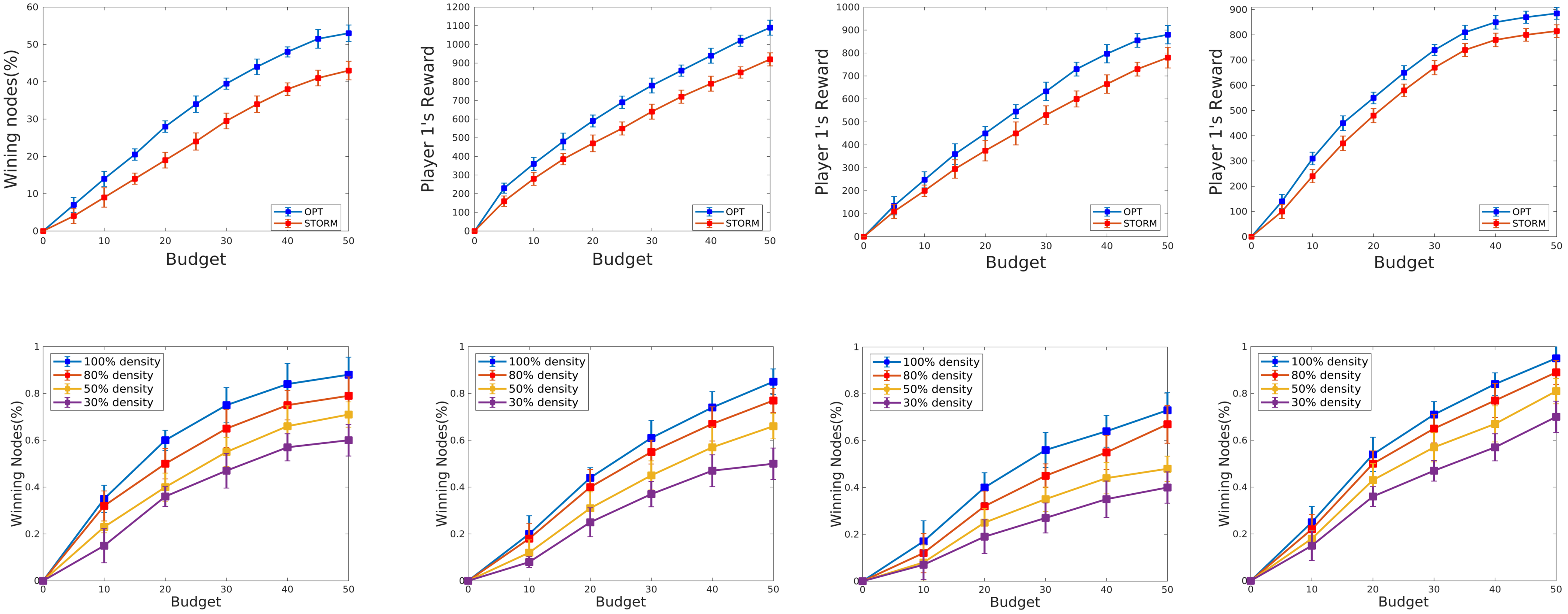
2) Evaluation Based on Different Topologies

3) Evaluation on Edge-weight Setting

- Case 1: uniform probabilities p on each edge
- Case 2: weight of the edges in the range of [0.1, 0.4] and [0.4, 0.7]. In addition, the weight for edges are randomly sampled from the normal distribution of U (0, 0.2) and U (0, 1).

4) Evaluation on Different Competing Strategies:


Player 1 's Reward
Budget
Randomly
STORM
OPT
0
5
10
15
20
25
30
35
40
45
50
55
60
Propagation of winning nodes(%
Difference of activated nodes
Competing Strategy
Degree
Weight
Centrality
MRIP
Random
Voting
STORM
OPT
20
40
60
80
100
120


Wining nodes(%)
Player 1's Reward
Player 1's Reward
Player 1's Reward
Budget
Budget
Budget
Budget
OPT
STORM
Winning Nodes(%)
Winning Nodes(%)
Winning Nodes(%)
Winning Nodes(%)
100% density
80% density
50% density
30% density
Budget
Budget
Budget
Budget

Thank you

Nadia.niknami@temple.edu