

# A Lightweight Message Dissemination Strategy for Minimizing Delay in Online Social Networks

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**Abstract**—Online Social Networks (OSNs) have attracted intensive attention for the reason that they provide users a convenient platform to share ideas, post events, and disseminate messages. Each OSN user commonly owns multiple social applications (Facebook, Google Plus and Twitter, etc.). They enjoy disseminating messages within one particular social application as well as forwarding interesting information to other social applications. In OSNs, some time-insensitive messages (disaster warnings, virus alerts, and search notices, etc.) are badly in need of being disseminated to specific users or applications as soon as possible. However, sudden message dissemination among users is bound to put a significant burden on network resources. Taking the dissemination cost into consideration, we propose a lightweight Message Dissemination strategy for Minimizing Delay in OSNs (MDMD), which first defines the user’s activeness according to the switch habit, among different social applications. Furthermore, depending on the activeness, an optimal user within each social application is selected to assist in disseminating the message. Simulation results show that, compared with other dissemination strategies, MDMD achieves the lowest average delay, and lower average hopcounts.

**Keywords**—OSNs, Social application, Lightweight, Minimizing delay, Activeness.

## I. INTRODUCTION

Online Social Networks (OSNs) [1] [2] have drawn great attentions in recent years for the reason that they provide various social applications, such as Facebook [3], Google Plus [4], Twitter [5], Wechat [6], Microblog [7], etc. Users can share humor, issue advertisements, make friends, and appoint activities through a variety of social application platforms [8]. They greatly enjoy disseminating interesting messages within the current social application, as well as forwarding the important messages among different social applications.

In this paper, we consider that there indeed exist some time-insensitive messages, which need to be disseminated not only within the current social application, but also to another specific social application as soon as possible [9]. For instance, disaster warning messages are bound to be disseminated to the users around a disaster area with minimal delay; virus alerts are necessarily notified to the gateway users as soon as possible; search notices must be quickly delivered to Uber, a social application used by taxi drivers. As far as we know, existing state-of-the-art literatures are lacking research focusing on minimizing message delivery delay among different social applications [10].

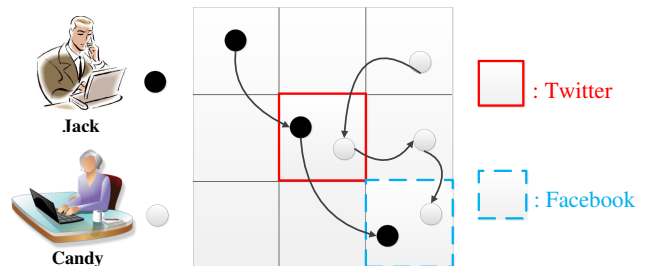


Fig. 1. An illustration in terms of message dissemination in Online Social Networks. Each grid represents a kind of social application, each circle represents a user, which could disseminate the message to any other user in the same social application. The users considered in this paper are within a social circle, which means that they are colleagues, classmates, or friends. Therefore, they could form a connection with each other in the same social application. Users switch among different social applications. It is necessary for us to determine an optimal user to disseminate the message in each social application for minimizing message dissemination delay.

In general, each OSN user owns multiple social applications. However, each user is merely active in one social application at any given moment. As can be seen in Fig. 1, Jack and Candy establish a connection through Twitter when they arrive at the red grid at the same time. Conversely, they could not establish a connection while in the different grids. After a period of time, Jack and Candy meet each other again in the Facebook social application. If Jack attempts to disseminate a message to the specific social application, he would like to choose an optimal user to help him forward the message as soon as possible. Therefore, it is not difficult to find that the switch habit among different social applications plays an important role in terms of minimizing delay. The active user which switches frequently is bound to assist in disseminating messages, while the fallow user, which stays on a social application for a long time, could not improve the dissemination performance.

In order to minimize dissemination delay, the problem changes into deciding an optimal user whose activeness is highest. In this paper, we propose a lightweight message dissemination strategy for minimizing delay in OSNs, which first defines user’s activeness according to the switch habit among different social applications. Subsequently, a dissemination strategy is decided on the basis of user’s activeness in order to minimize the dissemination delay, which always chooses the user with highest activeness to disseminate the message.

The main contributions of this paper are briefly summarized as follows:

- We define the user’s activeness in OSNs according to the switch habit among different social applications, where a higher switching frequency means higher activeness, and vice versa.
- According to the user’s activeness, a lightweight message dissemination strategy for minimizing delay is proposed in OSNs.
- We conduct extensive simulations based on the synthetic user’s activeness. The results show that, compared with other dissemination strategies, MDMD achieves the lowest average delay, and lower average hopcounts.

The remainder of the paper is organized as follows. We review the related work in Section II. The lightweight message dissemination strategy for minimizing delay in OSNs (MDMD) is presented in Section III. In Section IV, we evaluate the performance of MDMD through extensive simulations. We conclude the paper in Section V. Some proofs are presented in Appendix.

## II. RELATED WORK

### A. Maximizing Social Influence

Recently, an enormous amount of research has been devoted to selecting optimal users for message dissemination in order to maximize social influence. Kleinberg and Kempe *et al.* [1] prove that the optimization solution of selecting the most influential nodes is an NP-hard problem according to the analysis of several most widely-studied models in social networks. They are the first to provide a certifiable approximation guarantee for efficient algorithms. Based on the above work of Kleinberg, Chen *et al.* [11] show that computation complexity could be, in time, linear to the size of the graphs as long as network topologies are directed acyclic graphs (DAGs). Then they propose a scalable influence maximization algorithm.

### B. Controlling Network Burden

There are also plenty of research works focusing on reducing network burden through controlling key users, which are network gateway or community bridge users. Vojnović *et al.* [12] provide an analytical framework to evaluate the patching system performance, which accommodates various overlays utilizing the abstraction of a minimum broadcast curve, and also provides a filtering of scans across subnets. Zhu *et al.* [13] propose a counter-mechanism to restrict the flooding of a mobile worm as early as possible by patching an optimal gather of phones.

In this paper, we attempt to minimize the dissemination delay in OSNs, while the network burden problem is also considered. We prefer to make as few users as possible participate in the dissemination process. Therefore, in each social application, only one optimal user is selected to disseminate messages. In conclusion, a lightweight message dissemination strategy for minimizing delay is proposed in this paper.

## III. THE LIGHTWEIGHT MESSAGE DISSEMINATION STRATEGY FOR MINIMIZING DELAY IN OSNs

### A. Assumptions

Consider the following OSN environment; there are  $l$  users whose interests focus on  $n$  different social applications. The users shift among different social applications, and the residence time at each social application satisfies exponential distribution. Different users’ residence times satisfy different distributions, and the specific user’s residence times at different social applications obey the same distribution. The message is generated in the user with lowest activeness at an initial system time; each kind of message has only one copy and a given *TTL*, after which the message is no longer useful. Two users in the same social application could disseminate messages to each other. The main notations used in this paper are illustrated in Table I.

### B. Continuous-time Markov Model

1) *Definition of activeness:* In this section, we utilize two specific users ( $A$  and  $B$ ) to illustrate the continuous-time markov model. First of all, for the user  $A$ , we use  $X_t$  to express the state (social application being used) of user  $A$  at time  $t$ . It is not difficult to find that  $X_t \in e = \{e_1, e_2, \dots, e_n\}$ . This is due to the reason that, in the previous subsection, we assume that the time spent in each state takes non-negative values and obeys an exponential distribution. Therefore, we consider that  $X_t$  satisfies the continuous-time markov chains.

We define  $P_{ij}(t)$  as the probability that a user’s state of time 0 is  $i$ , and a user’s state of time  $t$  is  $j$ , which is shown as follows:  $P_{ij}(t) = P(X_t = j | X_0 = i)$ . Without loss of generality, we regard  $P(t)$  as the  $n * n$  matrix composed of  $P_{ij}(t)$ . We could generate the transition rate matrix  $P$  of continuous-time markov chains (as shown in Eq. 1) through the following formula:  $P = \lim_{t \rightarrow 0} \frac{P(t) - P(0)}{t - 0} = \lim_{t \rightarrow 0} \frac{P(t) - I}{t}$ , where  $I$  is the unit matrix.

$$P = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & a_{nn} \end{pmatrix} \quad (1)$$

The characters of matrix  $P$  are listed as follows:

- (1)  $a_{ii} < 0$  ( $\forall i$ ),  $\sum_{j=1}^n a_{ij} = 0$ ,  $a_{ij} \geq 0$  ( $\forall i \neq j$ ).
- (2) The user holding the message at social application  $e_i$  waits for a random time  $\tau_i$ , and then switches to  $e_j$  with the probability:  $q_{ij} = \frac{a_{ij}}{-a_{ii}}$ . The residence time satisfies exponential distribution, and therefore,  $\tau_i \sim \exp(-a_{ii})$ , indicating that  $\tau_i$  satisfies the exponential distribution with parameter  $-a_{ii}$ .
- (3) When the user holding the message stays in state  $i$ , it has an equal probability of switching from the current state to any other state, which means that the following equation is satisfied:  $q_{ij_1} = q_{ij_2}$  ( $\forall j_1, j_2 \neq i$ ).
- (4) The discrete states of  $X_t$  are expressed as:  $\overline{X_0}, \overline{X_1}, \dots, \overline{X_t}$ , which satisfy discrete-time markov chains.
- (5) The larger the  $-a_{ii}$  is, the higher the user’s activeness is. Similarly, the smaller the  $-a_{ii}$  is, the lower the user’s activeness is.

TABLE I. MAIN NOTATIONS USED THROUGHOUT THE PAPER

Notation	Explanation
$n$	Total number of social applications (i.e., number of grids) in OSNs
$e$	The set of different social applications, $e = \{e_1, e_2, \dots, e_n\}$
$X_t$	The user's state (social application being used) at time $t$ , $X_t \in e$
$P_{ij}(t)$	The probability that user's state of time 0 is $i$ , the state of time $t$ is $j$
$\tau_i$	The random residence time for a user in social application $e_i$
$c_k$	The parameter in exponential distribution of user $k$ 's residence times
$T$	The earliest time for two users to meet each other
$E_{ij}(T)$	The expected earliest time for two users to meet each other, with the condition that their initial states are $i$ and $j$
$\xi$	The first switch time (switch from one social application to another one) for any of the two users
$l$	The total number of users
$T_i$	The first meeting time between user $i$ and user $l$ (destination user)
$\lambda_{l,i}$	The parameter of $T_i$ 's exponential distribution, $\lambda_{l,i} = \frac{c_l + c_i}{n-1}$
$T^{(l)}$	The dissemination delay from the lowest priority user to highest priority user for a $l$ -users system
$r_l$	The expectation of $T^{(l)}$

In conclusion, each user has its corresponding transition rate matrix, where the negative diagonal  $-a_{ii}$  represents its activeness. Eq. 2 is a detailed example to further illustrate the user's activeness:

$$A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}, \quad (2)$$

where  $n = 3$  means that there are three social applications in OSNs.  $a_{ii} = -2$  indicates that the user's activeness is 2, and the expectation residence time satisfies:  $\tau_i \sim \exp(2)$ . In addition, the transition probability from state  $i$  to state  $j$  satisfies the following equation:  $a_{ij} = 1/2$  ( $\forall i \neq j$ ). Therefore, for this example, the user's residence time in each social application satisfies exponential distribution with parameter 2, and the user has the probability of  $1/2$  to switch from the current social application to any other one.

2) *The expectation time for the first meeting of A and B:* It is worth noticing that the expectation meeting time between user  $A$  and user  $B$  plays an important role for the reason that the research focuses on minimizing dissemination delay. Suppose that the state of user  $A$  at time  $t$  is  $X_t$ , and the state of user  $B$  at time  $t$  is  $Y_t$ . Simultaneously, according to the derivation in the previous subsection, the transition rate matrixes of user  $A$  and user  $B$  are shown in Eqs. 3 and 4, respectively. Moreover, on the basis of the character (1) of continuous-time markov chain's transition rate matrix,  $a$  and  $b$  are calculated through following two equations:  $a = \frac{c_1}{n-1}$ ,  $b = \frac{c_2}{n-1}$ . In addition, according to the character (3) of the transition rate matrix, user  $A$  and user  $B$  have the same probability  $\frac{1}{n-1}$  to switch from the current social application to another one. Their expectation residence times also satisfy exponential distributions with parameters  $c_1$  and  $c_2$ , respectively.

$$A = \begin{pmatrix} -c_1 & a & a & a \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & -c_1 \end{pmatrix} \quad (3)$$

$$B = \begin{pmatrix} -c_2 & b & b & b \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & -c_2 \end{pmatrix} \quad (4)$$

We define the earliest time for  $A$  and  $B$  to meet each other as  $T = \inf\{t \geq 0; X_t = Y_t\}$ , which means that the minimal nonnegative value  $t$  satisfying  $X_t = Y_t$ . Assume that the initial states of users  $A$  and  $B$  are  $X_t = i$  and  $Y_t = j$ , respectively. We define the notation of condition probability as:  $E_{ij}(\bullet) = E(\bullet | X_0 = i, Y_0 = j)$ , which indicates the expectation of earliest time for  $A$  and  $B$  to meet each other with the condition that  $X_t = i$  and  $Y_t = j$ . It is not difficult to find that our purpose changes into finding the value of  $E_{ij}(T)$ .

As previously mentioned, the state of user  $A$  is  $X_t$ , and the state of user  $B$  is  $Y_t$ . We define  $\xi^X$  as the user  $A$ 's first switching time from current social application to another one, and  $\xi^Y$  is the user  $B$ 's first switching time. We also define  $\xi = \min\{\xi^X, \xi^Y\}$  as the minimal switching time between user  $A$  and user  $B$ . According to the previous assumptions, Theorem 1 is achieved as follows.

**Theorem 1:**  $\xi^X \sim \exp(c_1)$ ,  $\xi^Y \sim \exp(c_2)$ , and then  $\xi \sim \exp(c_1 + c_2)$ .

The proof of Theorem 1 is shown in Appendix A, where we could achieve  $E_{ij}(\xi) = \frac{1}{c_1 + c_2}$ . The insight meaning of Theorem 1 is shown as follows. The expectation of user  $A$ 's first switching time is  $\frac{1}{c_1}$ , and the expectation of user  $B$ 's first switching time is  $\frac{1}{c_2}$ . However, the minimal expectation of switching time for either  $A$  or  $B$  is  $\frac{1}{c_1 + c_2}$ , which is lower than  $\frac{1}{c_1}$  and  $\frac{1}{c_2}$ . This is natural and reasonable. Based on Theorem 1, Theorem 2 is further obtained.

**Theorem 2:**  $\xi^X \sim \exp(c_1)$ ,  $\xi^Y \sim \exp(c_2)$ , the earliest time for  $A$  and  $B$  to meet each other ( $T = \inf\{t \geq 0; X_t = Y_t\}$ ) satisfies:  $T \sim \exp(\frac{c_1 + c_2}{n-1})$ .

The proof of Theorem 2 is shown in Appendix B. Theorem 2 shows the distribution of the expectation time for the first meeting of  $A$  and  $B$ , which has insightful meanings as follows. Due to the reason that  $T \sim \exp(\frac{c_1 + c_2}{n-1})$ ,  $E_{ij}(T) = \frac{n-1}{c_1 + c_2}$ . There are three variables:  $c_1$ ,  $c_2$ , and  $n$ . It is worth noticing that a larger  $c_1$  or  $c_2$  leads to a shorter first meeting time of  $A$  and  $B$ . In other words, a higher user activeness leads to an earlier meeting time, which makes sense. In addition, a larger  $n$  results in a longer first meeting time. That is to say, more social applications result in a longer meeting time.

In conclusion, in this section, we define the parameter of the exponential distribution obeyed by a user's residence time in each social application as user's activeness. According to a user's activeness, we achieve the expectation time for the first meeting between two users, which plays a major role in terms of making a message dissemination strategy, aiming to minimize delivery delay.

### C. Time-constant Activeness

We consider the online social network with  $l$  users, whose activeness (the parameter of exponential distribution regarding residence time in each social application) are different and time-constant. In other words, their priorities could be sorted from low to high. Assume that the activeness of  $l$  users in the network are  $c_1, c_2, \dots, c_l$ , while their priorities satisfy  $c_1 > c_2 > \dots > c_l$ . Without loss of generality, the user with lowest priority  $c_l$  is assigned as message holder, meanwhile, the destination user is considered with highest priority. There

is only one message holder in OSN at the same moment. The purpose is to make a dissemination decision for message holder in order to minimize delay.

$T_1, T_2, T_3, \dots, T_{l-1}$  are used to represent the first meeting times between user  $l$  and user  $1, 2, 3, \dots, l-1$ , respectively. According to the Theorem 2,  $T_i \sim \exp(\frac{c_l+c_i}{n-1})$  is achieved. In order to simplify the expression regarding the parameter of exponential distribution, we make the following symbolic representations:  $\lambda_{l,1} = \frac{c_l+c_1}{n-1}, \dots, \lambda_{l,l-1} = \frac{c_l+c_{l-1}}{n-1}$ .

The following two theorems are listed in order to further propose a message dissemination strategy for minimizing delay, when the user's activeness is time-constant.

**Theorem 3:**  $P(T_j < T_i, \forall i \neq j) = \frac{\lambda_{l,j}}{\lambda_{l,1}+\lambda_{l,2}+\dots+\lambda_{l,l-1}}$ .

*Proof* : consider the online social network with  $l$  users; the activeness of user  $i$  is  $c_i$ , and  $\lambda_{l,i} = \frac{c_l+c_i}{n-1}$ . The detailed proof process of Theorem 3 is shown in Eq. 5.

$$\begin{aligned} & P(T_j < T_i, \forall i \neq j) \\ &= \int_0^\infty \dots \int_0^\infty \chi_{t_j < t_i, \forall i \neq j} \prod_{i=1}^{l-1} (\lambda_{l,i} e^{-\lambda_{l,i} t_i}) dt_1 \dots dt_{l-1} \\ &= \int_0^\infty \lambda_{l,j} \prod_{i=1}^{l-1} e^{-\lambda_{l,i} t_j} dt_j = \frac{\lambda_{l,j}}{\lambda_{l,1} + \lambda_{l,2} \dots + \lambda_{l,l-1}} \quad (5) \end{aligned}$$

**Theorem 4:**  $E[T_j \chi_{T_j=\tau}] = \frac{\lambda_{l,j}}{(\lambda_{l,1}+\lambda_{l,2}+\dots+\lambda_{l,l-1})^2}$ .

*Proof* : the proof process of Theorem 4 is similar to the one in Theorem 3. Consider the OSN with  $l$  users as before, the proof process of Theorem 4 is shown in Eq. 6.

$$\begin{aligned} E[T_j \chi_{T_j=\tau}] &= \int_0^\infty \lambda_{l,j} t_j \prod_{i=1}^{l-1} e^{-\lambda_{l,i} t_j} dt_j \\ &= \int_0^\infty \frac{\lambda_{l,j}}{\sum_{i=1}^{l-1} \lambda_{l,i}} \sum_{i=1}^{l-1} \lambda_{l,i} t_j e^{-\sum_{i=1}^{l-1} \lambda_{l,i} t_j} dt_1 \\ &= \frac{\lambda_{l,j}}{(\lambda_{l,1} + \lambda_{l,2} \dots + \lambda_{l,l-1})^2} \quad (6) \end{aligned}$$

The purpose is to formulate a dissemination strategy for message holders in order to minimize dissemination delay. As shown in Table I, the dissemination delay from the lowest priority user to the highest priority user for a  $l$ -users system is defined as  $T^{(l)}$ . Similarly, the dissemination delay for a  $j$ -users system is  $T^{(j)}$ . The problem changes into calculating the expectation of  $T^{(l)}$ , which is shown in Eq. 7, where the calculation of  $E(T^{(l)})$  is divided into following two parts:  $E[T_j]$  and  $E(T^{(j)})$ .

$$E(T^{(l)}) = \sum_{j=1}^{l-1} \{E[T_j \chi_{T_j=\tau}] + P(T_j < T_i, \forall i \neq j) E(T^{(j)})\} \quad (7)$$

In order to simplify symbolic representation, we define that  $E(T^{(l)}) = r_l, E(T^{(j)}) = r_j$ . Then, when Eq. 8 is achieved, Theorem 5 is further proposed.

$$\begin{aligned} r_l &= \sum_{j=1}^{l-1} \left[ \frac{\lambda_{l,j}}{(\lambda_{l,1}+\lambda_{l,2}+\dots+\lambda_{l,l-1})^2} + \frac{\lambda_{l,j}}{\lambda_{l,1}+\lambda_{l,2}+\dots+\lambda_{l,l-1}} r_j \right] \\ &= \frac{1}{\lambda_{l,1}+\lambda_{l,2}+\dots+\lambda_{l,l-1}} + \frac{\sum_{j=1}^{l-1} \lambda_{l,j} r_j}{\lambda_{l,1}+\lambda_{l,2}+\dots+\lambda_{l,l-1}} \quad (8) \end{aligned}$$

**Theorem 5:**  $\lambda_{l,1} > \lambda_{l,2} > \dots > \lambda_{l,l-1}$ , if we exchange any pair of priorities  $\lambda_{l,i}$  and  $\lambda_{l,j}$ , then the  $r_l$  will get bigger.

The proof of Theorem 5 is shown in Appendix C, which illustrates that if we exchange any pair of initial priorities of  $\lambda$ , the expectation of dissemination delay will be longer. In other words, the best strategy for minimizing dissemination delay is to disseminate the message according to the priority:  $\lambda_{l,1} > \lambda_{l,2} > \dots > \lambda_{l,l-1}$ . Therefore, when the user's activeness is time-constant, we achieve the optimal dissemination strategy, which disseminates the message to the user of highest activeness in the current social application, in order to minimize dissemination delay.

## IV. PERFORMANCE EVALUATION

### A. Simulation Settings

To demonstrate the performance of the proposed MDMD, a multi-paradigm numerical computing environment MATLAB is employed in this paper. We carry out simulations with the synthetic user's activeness. Each user has a constant activeness, which means that the activeness never changes. Each result in the figure comes from the average value of 500 groups of simulations. The simulation time is set as 2000s. While a range of data is gathered from the simulations, on the premise that the delivery ratio is guaranteed to be 100% by the sufficient simulation time, we take the following two performance metrics into consideration.

- (1) Average delay, which is the average elapsed time of the successfully delivered messages.
- (2) Average hopcounts, which is the average forwarding number of the successfully delivered messages.

### B. Simulation Results under Time-constant Activeness

We assign the time-constant activeness to each user. The OSN environment composed of 10 users is considered. Their priorities of the activeness are scheduled as follows:  $c_1 = 1.0, c_2 = 0.9, c_3 = 0.8, \dots, c_9 = 0.2, c_{10} = 0.1$ . User 1 is regarded as the destination, and user 10 is assigned as the initial message holder. In order to compare with other dissemination strategies and prove the Theorem 5 through simulations, three different dissemination priorities are tested, and the simulation results are shown in Fig. 2. MDMD is the optimal strategy proposed in this paper; FC (FirstContact) is a random dissemination strategy, which disseminates the message to a random encounter; EO (ExchangeOrder) is a variation of MDMD, which randomly exchanges two users' priorities from the original order.

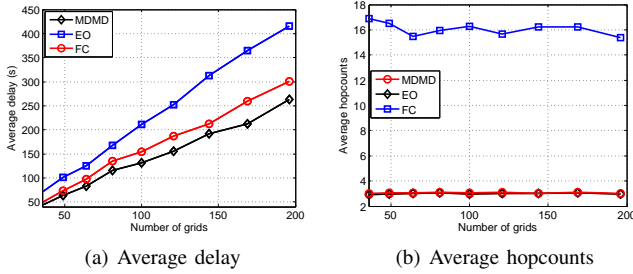


Fig. 2. Average delay and Average hopcounts as a function of number of grids when the user’s activeness is time-constant.

As can be seen in Fig. 2-(a), MDMD actually achieves the lowest average delay compared with the other two dissemination strategies, which further proves Theorem 5. Therefore, in order to minimize dissemination delay, the optimal solution is to disseminate the message to the user with the highest activeness. Fig. 2-(b) shows the changes in average hopcounts over the number of grids from 36 to 196. The simulation results show that MDMD achieves similar average hopcounts compared with EO, while far lower than the average hopcounts of FC.

## V. CONCLUSION

In OSNs, each user usually owns multiple social applications, such as Facebook, Google Plus and Twitter, etc. However, at the same moment, each user only could be active at one social application. They enjoy disseminating messages within the current social application, as well as forwarding interesting information to other social applications. There are some time-insensitive messages, which are in urgent need of being disseminated to specific users or applications as soon as possible. In order to solve the above problem, we propose a lightweight Message Dissemination strategy for Minimizing Delay in OSNs (MDMD), which first defines the user’s activeness according to the switch habit among different social applications. Furthermore, an optimal user with the highest activeness is selected to assist in disseminating the message. We conduct simulations in MATLAB with a synthetic user’s activeness. Simulation results show that, compared with other dissemination strategies, MDMD achieves lowest the average delay, and lower average hopcounts.

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## APPENDIX

### A. Proof of Theorem 1

Due to the reason that  $\xi = \min\{\xi^X, \xi^Y\}$ , therefore,  $\xi > t$  only when  $\xi^X > t$  and  $\xi^Y > t$ . Furthermore, the derivation result of Eq. 9 shows that  $\xi \sim \exp(c_1 + c_2)$ .

$$P(\xi > t) = P(\xi^X > t \cap \xi^Y > t) = P(\xi^X > t)P(\xi^Y > t) = e^{-c_1 t} e^{-c_2 t} = e^{-(c_1 + c_2)t} \quad (9)$$

### B. Proof of Theorem 2

Consider that  $X_0 = i$  and  $Y_0 = j$ ,  $\xi$  is the minimal switching time between users  $A$  and  $B$ . Therefore, after the switch of no matter  $A$  or  $B$ , the states of  $A$  and  $B$  change from  $i$  and  $j$  to  $g$  and  $k$ , which is expressed as  $(i, j) \rightarrow (g, k)$ .  $D$  is defined as the absorb state of  $A$  and  $B$ , which is shown as:  $D = \{(k, k), e_k \in e\}$ . When  $A$  and  $B$  enter the absorb state, it illustrates that user  $A$  and user  $B$  meet each other in the same social application. Moreover, it is not difficult to find that  $(i, j) \notin D$ . Otherwise, the expectation time for the first meeting of  $A$  and  $B$  is 0. In addition, after the first switch  $(i, j) \rightarrow (g, k)$ , if  $(g, k) \in D$ , then  $T = \xi$ . Otherwise,  $(g, k) \notin D$ , then user  $A$  and user  $B$  calculate the expectation of the first meeting time, afresh with the initial states of  $g$  and  $k$ . According to the above discussion, Eq. 10 is achieved.

$$E_{ij}(T) = E_{ij}(\xi) + \sum_{(g,k) \notin D} P((i, j) \rightarrow (g, k)) E_{gk}(T) + \sum_{(g,k) \in D} P((i, j) \rightarrow (g, k)) \cdot 0 \quad (10)$$

In order to simplify symbolic representation, we define  $\alpha = E_{ij}(T)$ , for the reason that there is no difference among all the social applications. Therefore,  $\alpha = E_{ij}(T) = E_{gk}(T)$ . We also use the discrete-time combination state  $Z_t$  to express the states of  $A$  and  $B$ :  $Z_t = (X_t + Y_t)$ , which means that  $\bar{X}_0 = i, \bar{Y}_0 = j$ , then  $\bar{Z}_0 = (i, j)$ . According to the Eq. 10 and  $E_{ij}(\xi) = \frac{1}{c_1+c_2}$ , Eq. 11 is achieved. Furthermore, the final equation in terms of  $\alpha$  is shown in Eq. 12

$$\alpha = \frac{1}{c_1 + c_2} + \sum_{(g,k) \notin D} P(\bar{Z}_1 = (g, k) | \bar{Z}_0 = (i, j)) \alpha \quad (11)$$

$$\alpha = \frac{1}{c_1 + c_2} \frac{1}{1 - \sum_{(g,k) \notin D} P(\bar{Z}_1 = (g, k) | \bar{Z}_0 = (i, j))} \quad (12)$$

Next, we consider the Kronecker sum [14]  $Z$  of matrixes  $A$  and  $B$ , which is the matrix sum defined by Eq. 13, where  $\oplus$  is Kronecker sum operation [14], and  $\otimes$  is Kronecker product operation [15]. In order to calculate  $\alpha$ , the matrix  $Z$  is used to achieve the transition probability from  $(i, j)$  to  $(g, k)$  in terms of the states of  $A$  and  $B$ .

$$Z = A \oplus B = A \otimes I + I \otimes B = (A \otimes I + I \otimes B)_{(i,j)(g,k)} = A_{ig} S_{jk} + S_{ig} B_{jk} \quad (13)$$

The problem changes into solving the matrix  $Z$ , where  $S_{jk}$  is defined as  $S_{jk} = \begin{cases} 1 & j = k \\ 0 & k \neq k \end{cases}$ . Similarly,  $S_{ig} = \begin{cases} 1 & i = g \\ 0 & i \neq g \end{cases}$ . The following four cases are considered in order to calculate matrix  $Z$ .

- Case 1: if  $(g, k) = (i, j)$ , then  $Z = A_{ig} + B_{jk} = -(c_1 + c_2)$
- Case 2: if  $(g, k) \neq (i, j)$  and  $j = k, i \neq g$ , then  $Z = A_{ig} = \frac{c_1}{n-1}$
- Case 3: if  $(g, k) \neq (i, j)$  and  $j \neq k, i = g$ , then  $Z = B_{jk} = \frac{c_2}{n-1}$
- Case 4: other situations,  $Z = 0$

Based on the above four cases, Eq. 14 is obtained. The value of  $\alpha$  is further calculated through Eq. 15.

$$\begin{aligned} & \sum_{(g,k) \notin D} P(\bar{Z}_1 = (g, k) | \bar{Z}_0 = (i, j)) \\ &= \frac{(n-2)c_1}{n-1} \frac{1}{c_1 + c_2} + \frac{(n-2)c_2}{n-1} \frac{1}{c_1 + c_2} \\ &= \frac{n-2}{n-1} \end{aligned} \quad (14)$$

$$\alpha = (n-1) \frac{1}{c_1 + c_2} \quad (15)$$

Combining Eq. 10 and Strong Markov property [16], we get the following equation:

$$\begin{aligned} E_{ij}(e^{-sT}) &= E_{ij}(e^{-s\xi}) \sum_{(g,k) \notin D} P((i, j) \rightarrow (g, k)) E_{gk}(e^{-sT}) + \\ & E_{ij}(e^{-s\xi}) \sum_{(g,k) \in D} P((i, j) \rightarrow (g, k)). \end{aligned} \quad (16)$$

Where  $E_{ij}(e^{-s\xi})$  satisfies Eq. 17.

$$\begin{aligned} E_{ij}(e^{-s\xi}) &= \int_0^\infty e^{-st} (c_1 + c_2) e^{-(c_1+c_2)t} dt \\ &= \frac{c_1 + c_2}{c_1 + c_2 + s} \end{aligned} \quad (17)$$

In order to simplify symbolic representation, we define that  $\beta = E_{ij}(e^{-sT})$ . Combining  $\beta = E_{ij}(e^{-sT})$  and Eq. 16, we get the equation of  $\beta$  as Eq. 18

$$\beta = \frac{c_1 + c_2}{c_1 + c_2 + s} \left[ \left(1 - \frac{1}{n-1}\right) \beta + \frac{1}{n-1} \right] \quad (18)$$

Next, through solving Eq. 18, we obtain the final expression of  $\beta$ , which is shown in Eq. 19. Therefore,  $T \sim \exp(\frac{c_1+c_2}{n-1})$  is achieved through a method similar to that of Eq. 17. Theorem 2 is proved.

$$\beta = \frac{\frac{c_1+c_2}{n-1}}{\frac{c_1+c_2}{n-1} + s} \quad (19)$$

### C. Proof of Theorem 5

First of all, we attempt to prove following result:  $r_k > r_{k-1}, \forall k \in (1, 2, 3 \dots, l)$ . It is known that  $r_k$  represents the delivery delay for a message from  $c_k$  to  $c_1$ , and  $c_1 > c_2 > \dots > c_k$ . There exist two situations according to the difference of the first dissemination step. First situation: the first step, where the message is disseminated to  $c_{k-1}$ , assuming that the time of the first step is  $t'$ , and the second step, the message is disseminated from  $c_{k-1}$  to  $c_1$ , and the delay is  $r_{k-1}$ . Therefore, the total delay from  $c_k$  to  $c_1$  in the first situation is  $r_k = r_{k-1} + t' > r_{k-1}$ . The second situation: in the first step, the message is not disseminated to  $c_{k-1}$ , therefore, the system is assumed to be a  $r_{k-1}$  system, where  $c_1 > c_2 > \dots > c_{k-2} > c_k$ . Due to the reason that  $c_{k-1} > c_k$ , the activeness level for user  $k-1$  is higher than that of user  $k$ . Therefore  $r_k > r_{k-1}$  can also be achieved in the second situation. In conclusion,  $r_k > r_{k-1}, \forall k \in (1, 2, 3 \dots, l)$ . Based on the above proof, we get the conclusion that  $r_1 < r_2 < r_3 < \dots < r_l$ , and the  $r_l$  satisfies:

$$r_l = \frac{1}{\lambda_{l,1} + \lambda_{l,2} + \dots + \lambda_{l,l-1}} + \frac{\sum_{j=1}^{l-1} \lambda_{l,j} r_j}{\lambda_{l,1} + \lambda_{l,2} + \dots + \lambda_{l,l-1}} \quad (20)$$

if we exchange  $\lambda_{l,i}$  and  $\lambda_{l,j}$ , the variation of  $r_l$  is  $(\lambda_{l,i} r_j + \lambda_{l,j} r_i) - (\lambda_{l,j} r_j + \lambda_{l,i} r_i) = (r_j - r_i)(\lambda_{l,i} - \lambda_{l,j}) > 0$

Therefore,  $\lambda_{l,1} > \lambda_{l,2} > \dots > \lambda_{l,l-1}$  is actually the optimal priority order, and we get the conclusion that: *Theorem 5* is proved.