A Blockchain-Powered Data Market for Multi-User Cooperative Search

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***Abstract*—Cloud computing provides a feasible solution to data outsourcing, and hence forming a cloud-based data market, where data users buy data from owners through querying cloud servers. However, it also incurs new privacy and security problems, as data is under a centralized third-party instead of the data owner's direct control. Existing data markets are also questioned on their inflexible and opaque pricing, where the value of data ownership and the cost of query searches are mixed. In this paper, we consider blockchain-based storage as a better choice to ensure safe data outsourcing since data is spread out across many data points. We propose an Ethereum-based data market that provides distributed storage and correct remote data search. We design a new pricing model, where each query will be charged by two parties: owner (paid for providing his data) and miner (rewarded by performing query searches). We study a new cooperative search scheme through a proxy to reduce cost on the user side. Given that each user query is charged based on its number of keywords, then a cooperative search can reduce user-side cost by combining multiple queries into a group so that overlapped keywords will only be charged for one time. To ensure user QoE, a combined query should not be significantly larger than any of its original queries in terms of the number of keywords. The total price is based on the total number of keywords in all groups. Since it is a cooperative model with shared resources, we also study various incentive properties on the user side, yielding a cost sharing mechanism to split joint cost in a truth-revealing and fair manner. We further extend our market with a set of substitute data owners and propose a double auction mechanism to match users and owners based on their requirements. Experiments have been conducted on real query trace to demonstrate the effectiveness of our proposed scheme.**

***Index Terms*—Blockchain, cooperative search, cost model, cost sharing, double auction, grouping strategy.**

I. Introduction

Currently, data is considered as one of society's most valuable resources, and the resulting market further monetizes data. Without question, data has become a tradeable commodity in our society. Most data markets, *e.g*. Amazon Athena and Xignite, are online, making it convenient to publish and exchange data. Currently, those online markets are tightly coupled with Cloud Computing (CC) paradigm. CC offers a cost-efficient solution for trading data and provides value-added services that help derive data products. Data owners usually rent resources from a certain CC platform to store and manage their data as well as answer data users' queries. However, centralized CC platforms also give rise to privacy and security issues in data storage and search. Thus, decentralized storage services have come into being to alleviate these concerns. Blockchain techniques [2] are widely used to guarantee data integrity and provide scalability for handling big data. Meanwhile, smart contract [3] can be leveraged to perform remote searches automatically, which ensures the correctness and immutability of search results. Unfortunately, lots of existing works focus on the single-user setting, where a database is queried only through its owner. As in real data markets, a data owner makes profit by sharing his data with legitimate users. A more complex multi-user setting should be discussed in the perspective of decentralized storage.

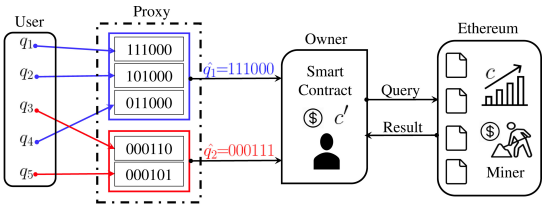

Fig. 1: Given a database with six keywords, five queries q_1, q_2, q_3, q_4, q_5, each being a six-bit binary string with 1 for search and 0 for not search, are issued from users to retrieve information.

In this paper, we propose an Ethereum-based data market, where many data-related services, *e.g*. storage, search and trades, can be provided for both data owners and users. Ethereum [4] is a decentralized computing platform that combines blockchain technique and smart contract[3]. Thus, it guarantees reliable data storage and correct remote search. The proposed data market is shown in Fig. 1 and consists of three basic roles. As a data provider, an *owner* profitably shares his database by allowing a third-party called *users* to query his database. Different from those CC-based data markets, private data is no longer uploaded to a central provider. Instead, an Ethereum-based data market provides a decentralized fashion for data storage and management. An owner can either send his small-size information to the Ethereum blockchain [3] for the convenience of searching, or distribute his large data in an off-blockchain storage [5], *e.g*. IPFS [6], with a pointer to the data on the distributed ledger of blockchain. We assume all data is encrypted under a searchable symmetric encryption (SSE) scheme before being outsourced, so that its confidentiality can be preserved while still allowing query-based searches. Users are data consumers and are willing to retrieve information from a designated database with some payment. We assume that users send queries directly to a corresponding owner, then the

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owner issues search transactions as if he is querying. As data searchers, Ethereum *miners* make money by executing search functions according to smart contracts.

Thus, query pricing in such a decentralized data market should be decided by owners and miners jointly. Each user query is charged for two parts: one from the data owner at a pre-set price based on data value, and the other from miners based on their workload, which is a pay-as-you-go mode. Miners' workload, *i.e.*, how much computation power is consumed to perform a query, is traceable and transparent in Ethereum, due to its gas system. Besides, a query's computation consumption and the corresponding search delay, as well as its cost, tend to be proportional to its keyword number.

As a cheap query price is desirable by each user, *cooperative search* can be a good approach to save total cost, hence decreasing individual payments. To illustrate, let us consider an example in Fig. 1. There are five users that want to search over the same database which includes six keywords in total. The owner will charge a cost of c' per query, and miners will charge a cost of c per keyword. Thus, each query will be separately charged by the owner and miners. For example, the cost of query $q_1 = 111000$ (a six-bit binary string for six keywords, where 1 represents that the corresponding keyword is selected and 0 represents not being selected) is c' for the owner plus $3c$ due to 3-keyword search in Ethereum. Without cooperation, the overall cost of these queries is $5c' + 11c$ (5 users and 11 accumulative keywords). To save cost, their queries can be grouped. Among all the grouping strategies, we briefly mention two here: (1) a *cost-saving-based* strategy without considering delay constraints. Five users are grouped together, and their combined query is 111111 at a total cost of $c' + 6c$. Search latency largely increases for each user. (2) a *delay-tolerant-oriented* strategy (as is shown in Fig. 1) that groups q_1, q_2, q_4 in G_1, and q_3, q_5 in G_2. Thus, the total cost for all users is $2c' + 6c$, and any user at most waits an additional 1-keyword search time. This example reflects a tradeoff between delay constraint and cost efficiency.

To ensure the quality of experience, users are considered to be limited delay-tolerant. We design a user-biased cooperative search scheme, that facilitates group formation among users driven by cost savings, subject to a uniform delay constraint all users agree upon. Users submit their individual queries and delay constraints to a front *proxy*. The proxy gathers users with similar delay constraints into a set, and matches users from the same set as search groups in order to minimize the overall cost subject to the delay constraint, *i.e.*, the number of 1s in each group query should not exceed a given number. After grouping, the data owner receives combined queries from the proxy and issues search requests to Ethereum. This cooperative search scheme improves a data owner's processing capacity by reducing the query number.

In a cooperative model, cost sharing must be regulated in a truth-revealing and fair manner. Truth-revealing helps avoid free-riding users who want to get some information without payment, and fairness promotes a stable and long-term cooperation among users. For example, we consider a common cost sharing mechanism, which equally distributes group cost among its members. After applying it to the grouping result in Fig. 1, q_1, q_2, and q_4 will be equally charged with $(c' + 3c)/3$. It seems unfair because q_1 has more keywords in its original form compared with q_2 and q_4. In our paper, we design a keyword-based cost sharing mechanism according to original queries, which yields some desirable properties like group-strategyproofness and sharing incentive. In Fig. 1, the cost $2c'$ paid to the owner will be equally distributed among 5 users, each of whom is responsible for $2c'/5$. The total cost of searching for the first keyword is c, equally shared by q_1 and q_2. The last keyword also costs c, which is only borne by q_5, since there is no other user querying this keyword.

In fact, a data market may consist of many owners with functionally-similar data so that users can choose any of them based on owners' prices and data quality. We further extend our model to include such a multiple-substitute-data-owner scenario. We provide a decentralized double auction method to match users and owners by utilizing Ethereum smart contracts. The contributions of this paper are summarized as follows:

- Extending from the single-user setting in decentralized storage, we propose an Ethereum-based data market.
- A user-biased, limited-delay-tolerant cooperative search scheme driven by cost savings is designed to maximize social welfare on the user side.
- We formulate an n-query-grouping problem as a set partition problem, prove it as NP-hard, and solve it through approximation with guaranteed bounds, as well as an efficient projected gradient descent method.
- A cost sharing mechanism is provided to fairly split the total payment among all participating users. This mechanism guarantees several desirable properties, *e.g.* group-strategyproofness and sharing incentive.
- We further extend our market with a set of substitute data owners and propose a double auction mechanism to match buyers and owners based on their requirements.
- Extensive evaluations on real query traces AOL demonstrate the effectiveness of our cooperative scheme. A small testbed of an Ethereum-based data market is also implemented to show the relationship between the number of keywords and the search delay by the miners.

II. Related Work

A. Ethereum and Smart Contract

As a blockchain-based decentralized platform, Ethereum [4] also supports smart contract. A smart contract can be viewed as a self-enforced computer program. Its execution is automatic when meeting the predefined conditions. The Ethereum blockchain technique ensures that after being created and deployed, each smart contract cannot be modified forever. Each valid transaction will trigger parts of codes in some smart contract(s) executed in each miner's Ethereum Virtual Machine (EVM) and will change states of corresponding smart contract(s). Mining new blocks requires a miner to correctly process transactions and successfully solve crypto puzzles. Since an approved transaction should be validated by the whole network, its corresponding execution results must reach consensus among all miners and no one can modify it. This guarantees correctness and immutability of execution results.

B. Online Data Markets

Traditional online data markets allow data owners to pose their data information on some digital platforms [7, 8] so that buyers (users) can match them based on their requirements. Usually, buyers have to buy the entire database from a seller, then download and query it offline. This is an inefficient solution for buyers. On cloud-based data markets [9, 10], data owners upload their data to the cloud and make money by sharing it with users. Users pay to obtain what they want through a query interface instead of buying the entire database. Cloud providers are rewarded by providing services, *e.g.* storage and search. Our Ethereum-based data market is similar to cloud-based data markets, where users pay for querying, while owners and miners earn by providing data and services, respectively. There exist some works designing data market on top of Blockchain such as [11–14], since blockchain can enable secure data trading and search [15–18].

C. Cost Models

Traditional online markets take a one-time payment set by data owners. For data markets based on cloud computing, either data owners [9] or cloud providers [10] can be sellers, and they provide term-based offers such as monthly subscriptions. Some cloud providers use a pay-as-you-go mode. Each query is charged based on bytes of scanned data [19]. On the Ethereum-based data market, owners and miners are all sellers, jointly charging users. Most online data markets adopt a unilateral cost model [20], while the cost model can also be bilateral, such as auction, in a more complex situation.

D. Cooperative Search

The cooperative keyword-based search [21] scheme has been proposed in [22] by grouping all received queries together within a fixed timeout to achieve privacy. In [23], authors equally divide queries into a fixed number of groups to achieve k-anonymity and load balancing. Our work also deals with query grouping problems, while focusing on user-side cost saving and search delay guarantee.

E. Cost Sharing Schemes

A good cost sharing mechanism will distribute shared cost among users in a truth-revealing and fair manner [24]. The current cost sharing mechanism is group-based [23]. Our proposed mechanism is keyword-based, which guarantees group-strategyproofness and sharing incentive.

III. Preliminaries

A. Scheme Overview

There are two stages in our proposed cooperative search scheme, and four entities are involved. Fig. 1 shows the system overview. The first stage includes three different entities: users, a proxy, and a data owner. Although we assume a single data owner and proxy, it can be easily extended to multiple owners and proxies. All user queries and the corresponding delay constraints are directly sent to the proxy. We assume that users are exposed to the proxy by using credentials. Each time a user submits a query, he is assigned to a temporary credential that will be changed in the next submission. Thus, users are unlinkable to their queries as the proxy only knows their temporary credentials, *i.e.*, no user privacy leaking to the proxy. The proxy partitions users into different sets based on the delay constraints. How to gather users so that queries in the same set have similar delay constraints is out of the scope of this paper. However, this topic is quite similar to the task scheduling problems in the data center where many papers can be referred to [25–27]. Each set is supposed to be sufficiently large, and thus we treat them separately. In each set, the minimal value of all users' delay constraints is treated as the set delay upper bound. Given an n-query set, the main function of the proxy is to run our grouping strategies, which classify n queries into k groups (k is a variable) and send those combined queries to the data owner. In the second stage, a data owner issues search transactions to Ethereum, based on queries received from his proxy. Then miners execute related search functions and obtain query results. We also provide a cost sharing mechanism so that n users can share their total costs in a fair way.

B. System Workflow and Data License

All interactions between the above entities can be concluded as follows: (1) a user sends his individual query to a designated proxy; (2) a proxy partitions all received queries based on a uniform delay constraint and forwards them to the corresponding data owner; (3) a data owner issues search transactions for each combined query he receives as if he himself is querying; (4) all miners compete for mining a new block including executing corresponding search transactions. The query result will directly return to the proxy. The proxy will filter the combined results and then send them back based on the users' original queries. To ensure a legal right to access those results, each user has to pay an extra money to get a data license from the data owner. We assume the cost for an extra data license is much cheaper compared with the original access cost so that the group-buying is always cost-efficient.

C. Design Goals

We summarize our design goal into three parts. First, we want to design effective grouping strategies in order to achieve *social optimum*: minimizing the total cost among n users, and *latency limitation*: each user can be guaranteed to retrieve desired information within their uniformly-agreed delay tolerance, simultaneously. Second, we want to design a cost sharing mechanism among n users, which satisfy *group-strategyproofness*: each user should reveal his individual query request in a truthful way even on the condition where collusion is permitted, since lying provides no benefit to his interest, and *sharing incentive*: each user should achieve individual cost reduction if their total cost gets reduced. Third, we would like to implement an Ethereum testbed to verify the practicality of our proposed search scheme, and the relationship between the keyword number and the search delay.

TABLE I: Summary of Notations.

Symbol	Description
d	number of keywords in the dictionary
w_i	the i-th keyword in the dictionary
n	number of queries, where $n \geq 2$
Q	n-query set
G_i	group i, which is a subset of Q
$\|G_i\|$	number of queries in G_i
q_i	query from user i, in the form of a binary string
$\|q_i\|$	number of keywords in q_i
$\hat{q}_i$	combined query of G_i
$\|\hat{q}_i\|$	number of keywords in $\hat{q}_i$
k	number of groups
P_i	a partition over Q with i non-overlapping groups
c	cost of miner searching for a keyword
c'	cost of a data owner
$\gamma c'$	cost of a license from the data owner, where $0 \leq \gamma \ll 1$

IV. Grouping strategy

A. Notation and Cost Model

Our grouping strategy will be executed on a set of n queries, $Q = \{q_1, q_2, \cdots, q_n\}$, which are issued from different users over the same database. Corresponding notations are listed in Table I.

The cost $C(q)$ of a query q consists of two parts: c' is charged by a corresponding owner due to his contribution on data and $c \cdot |q|$ is paid to a miner in search of information. If a query result will be shared with others, extra data licenses are needed. Each result access requires a license at a cost of $\gamma c'$, where γ is much less than 1.

1) Cost without Grouping: The n queries in the set Q are individually executed and their total cost is the accumulation of their individual costs.

$$\sum\nolimits_{i=1}^{n} C(q_i) = c' \cdot n + c \cdot \sum\nolimits_{i=1}^{n} |q_i| \tag{1}$$

2) Cost with Grouping: Another way to execute Q is to create a single group G that contains all n queries and execute it as a single query $\hat{q} = |q_1 \vee \cdots \vee q_n|$. Note that, each user needs to buy a data license in this case.

$$C(\hat{q}) = c' \cdot (1 + n\gamma) + c \cdot |\hat{q}| \tag{2}$$

We can determine if grouping is beneficial by comparing Eq. (1) with Eq. (2).

$$\sum\nolimits_{i=1}^{n} C(q_i) - C(\hat{q}) = c'(n - 1 - n\gamma) + c(\sum\nolimits_{i=1}^{n} |q_i| - |\hat{q}|) \tag{3}$$

Even in the worst case, where there are no overlapping keywords among n queries, *i.e.*, $\sum_{i=1}^{n} |q_i| = |\hat{q}|$, Eq. (3) ≥ 0 still holds. It is obvious that grouping is always beneficial. Thus, greedily grouping all queries into a combined query is always a socially optimal choice, if no user has a constraint on search delay.

B. Problem Formulation

Given $Q = \{q_1, q_2, ..., q_n\}$, let α be the pre-agreed maximal search delay, measured in the numbers of keywords among n users. That is, each user is willing to wait for at most α-keyword search time, whatever their original keyword numbers before grouping. An optimal grouping problem is defined as:

Problem 1 (Optimal Query Grouping, OQG)**.** *Given a set of queries $Q = \{q_1, q_2, ..., q_n\}$, group the n queries into k non-overlapping groups $G_1, G_2, ..., G_k$ (k is a variable), such that the number of keywords in each combined query is no more than α and the overall cost of all groups is minimized.*

C. Problem Hardness

Theorem 1. *The OQG problem is NP-hard.*

Proof. The Set Partitioning (SP) problem, known to be NP-hard, can be reduced to the OQG problem. The SP problem is expressed as: given a set of n positive integers $\{a_1, a_2, \cdots, a_n\}$ and an integer A, where $\forall i \in [1, n],\ a_i \leq A$, such that $\sum_{i=1}^{n} a_i = 2A$, decide if this set can be partitioned into two subsets with the same sum A. We can construct every instance of the SP problem as a valid instance of the OQG problem as follows. Let α, the upper limit of the keyword number in each combined query, be equal to A, and d, the number of keywords in the dictionary, be $2A$. Each a_i is mapped to a query q_i. We define the number of keywords for each query $|q_i|$ as a_i. We also assume that there is no overlapping among all queries. This is a valid instance of the OQG problem.

An optimal solution to the OQG problem with two groups exists, if and only if there exists a partition in the original SP problem. Obviously, merging n queries into a single group G is infeasible because the combined query $\hat{q}$ contains $2A$ keywords, exceeding the upper limit. Hence, at least one query should be removed from G. According to our previous analysis, for any group G_i in the optimal solution, its cost is $C(\hat{q}_i) = c' \cdot (1 + n\gamma) + c \cdot |\hat{q}_i|$. We minimize $C(\hat{q}_i)$ over all k groups, but $c \sum_{i=1}^{k} |\hat{q}_i|$ is constant among all grouping strategies, hence our problem is equivalent to minimizing the number of groups. If the OQG problem has an optimal solution with two groups G_1 and G_2, then there exists a partition of n queries into two sets, such that $\sum_{q_i \in G_1} |q_i| = \sum_{q_j \in G_2} |q_j| = \alpha$. This partition is definitely an optimal solution to the SP problem. On the other hand, if the original SP problem has an equal-sum partition, the OQG problem also has an optimal strategy with two groups, since three groups would yield more cost. We can conclude that the OQG problem is NP-hard. □

D. Grouping Strategy

Since the OQG problem is NP-hard, we solve it using linear programming relaxation and greedy algorithms. No matter what the grouping result is, the cost of the data license is fixed as $n\gamma c'$; we will ignore this part in the rest of the paper.

1) Mathematic Relaxation: First, we give the matrix representation of the above problem. Since each user has a query string with length d, we define an $n \times d$ matrix $\mathbf{Q}$ as representing all queries from n users. Let $\mathbf{Y} \in \{0,1\}^{n \times n}$ denote the grouping result, then each (i, j)-th element of $\mathbf{Y}$ takes either 0 or 1. $Y_{ij} = 1$ means query q_i is classified into group G_j. The matrix representation is shown below.

$$\arg\min_{\mathbf{Y}} \quad c' \cdot tr(\delta(\mathbf{Y}^T\mathbf{E})) + c \cdot tr(\mathbf{E}^T\delta(\mathbf{Y}^T\mathbf{Q})) \tag{4a}$$

$$\mathbf{Y} \in \{0,1\}^{n \times n},\ \delta(\mathbf{Y}^T\mathbf{Q})\mathbf{e} \leq \alpha\mathbf{e},\ \mathbf{Y}\mathbf{e} = \mathbf{e} \tag{4b}$$

where tr is the trace operator which sums up diagonal elements of a given matrix. $\mathbf{E}$ and $\mathbf{e}$ denote an all-1's matrix and

$$\mathbf{Q}_{5\times 6}=\begin{bmatrix}1&1&1&0&0&0\\1&0&1&0&0&0\\0&1&1&0&0&0\\0&0&0&1&1&0\\0&0&0&1&0&1\end{bmatrix} \qquad \mathbf{Y}_{5\times 5}=\begin{bmatrix}1&0&0&0&0\\1&0&0&0&0\\0&1&0&0&0\\1&0&0&0&0\\0&1&0&0&0\end{bmatrix}$$

(a) (b)

$$\delta(\mathbf{Y}^T\mathbf{E})=\delta\left(\begin{bmatrix}1&1&0&1&0\\0&0&1&0&1\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\times\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}\right)=\begin{bmatrix}1&&&&\\&1&&&\\&&0&&\\&&&0&\\&&&&0\end{bmatrix}$$

(c)

$$E^T\delta(\mathbf{Y}^T\mathbf{Q})=\begin{bmatrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{bmatrix}\times\begin{bmatrix}1&1&1&0&0&0\\0&0&0&1&1&1\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}=\begin{bmatrix}1&&&&&\\&1&&&&\\&&1&&&\\&&&1&&\\&&&&1&\\&&&&&1\end{bmatrix}$$

(d)

Fig. 2: Matrix representation of queries and grouping result (We don't fill all elements in some matrices for saving space).

an all-1's vector, respectively, and both of their sizes can be adjusted to fit the context. And $\delta(\cdot)$ is an indicator function such that $\delta(c)=1$ if c is non-zero and $\delta(c)=0$, otherwise. The value of (i,j)-th element in matrix $\mathbf{Y}^T\mathbf{Q}$ represents how many times the keyword w_j is queried among all members in the group G_i. Since each overlapping keyword in a group only needs querying once, $\delta(\mathbf{Y}^T\mathbf{Q})$ indicates combined queries for all groups. Thus, $tr(\mathbf{E}^T\delta(\mathbf{Y}^T\mathbf{Q}))$ calculates the total keyword number of all combined queries. Similarly, $tr(\delta(\mathbf{Y}^T\mathbf{E}))$ indicates the true group number.

We use an example with five queries to better explain each term in Eq. (4a). Those five queries q_1, q_2, q_3, q_4, q_5 are represented as a matrix $\mathbf{Q}$ in Fig. 2(a), where the i-th row represents q_i. The grouping result $\mathbf{Y}$ is provided in Fig. 2(b), indicating that there are two groups, q_1, q_2, and q_4 in G_1, and q_3, q_5 in G_2. Thus, the number of groups can be calculated using the expression $tr(\delta(\mathbf{Y}^T\mathbf{E}))=2$ based on Fig. 2(c). Besides, the combined queries of G_1 and G_2 are $\hat{q}_1=111000$ and $\hat{q}_2=000111$, which are consistent with its matrix representation $\delta(\mathbf{Y}^T\mathbf{Q})$ shown in Fig. 2(d). Hence, the total keyword number over all combined queries $tr(\mathbf{E}^T\delta(\mathbf{Y}^T\mathbf{Q}))=6$ is equal to $|\hat{q}_1|+|\hat{q}_2|$.

Our objective is to find a grouping strategy $\mathbf{Y}$, which will result in a minimal overall cost for n queries, as Eq. (2) shows, while each combined query has no more than α keywords ($\delta(\mathbf{Y}^T\mathbf{Q})\mathbf{e}\leq\alpha\mathbf{e}$) and any original query only belongs to a group ($\mathbf{Ye}=\mathbf{e}$). Since it is an NP-hard problem, we consider a relaxation. According to [22], given a large constant number β, *e.g.* $\beta=10,\ 20,\ 30$, the indicator function over the matrix $\mathbf{Y}^T\mathbf{Q}$, *i.e.*, $\delta(\mathbf{Y}^T\mathbf{Q})$, can be approximated with a smooth function, $\mathbf{E}-e^{(-\beta\mathbf{Y}^T\mathbf{Q})}$, and the indicator function over the matrix $\mathbf{Y}^T\mathbf{E}$, *i.e.*, $\delta(\mathbf{Y}^T\mathbf{E})$, can be approximated with a smooth function, $\mathbf{E}-e^{(-\beta\mathbf{Y}^T\mathbf{E})}$. With the above relaxations, we transfer the integer matrix $\mathbf{Y}$ into the continuous domain. Then, we get a relaxed version of the OQG problem below:

$$\arg\min_{\mathbf{Y}} \quad c'\cdot tr(\mathbf{E}-e^{-\beta\mathbf{Y}^T\mathbf{E}})+c\cdot tr(\mathbf{E}^T[\mathbf{E}-e^{-\beta\mathbf{Y}^T\mathbf{Q}}])$$

TABLE II: Examples of 7 User Queries.

Queries	Content
q_1	11010000
q_2	00001101
q_3	11000000
q_4	00000111
q_5	00001100
q_6	00000011
q_7	10000000

(a) Example One.

Queries	Content
q_1	11010000
q_2	00001101
q_3	11000000
q_4	00000111
q_5	00001100
q_6	10000001
q_7	00110000

(b) Example Two.

$$\mathbf{Y}\in[0,1]^{n\times n},\ \mathbf{Ye}=\mathbf{e},\ e^{-\beta\mathbf{Y}^T\mathbf{Q}}\mathbf{e}\geq(d-\alpha)\mathbf{e}$$

which equals to its dual problem as below:

$$\arg\max_{\mathbf{Y}} \quad c'\cdot tr(e^{-\beta\mathbf{Y}^T\mathbf{E}})+c\cdot tr(\mathbf{E}^Te^{-\beta\mathbf{Y}^T\mathbf{Q}}) \tag{5}$$
$$\mathbf{Y}\in[0,1]^{n\times n},\ \mathbf{Ye}=\mathbf{e},\ e^{-\beta\mathbf{Y}^T\mathbf{Q}}\mathbf{e}\geq(d-\alpha)\mathbf{e}$$

The objective of Eq. (5) is to maximize a convex function over multiple variables with nonlinear constraints. We apply the interior-point approach, transforming the original inequality-constrained problem into a sequence of equality constrained problems. A logarithmic barrier function with a dynamic coefficient μ is constructed and added to the original objective function to remove all inequality constraints. This algorithm has a two-level iteration. The outer level iterates over the coefficient μ, while the inner level optimizes the augmented objective function using the Newton method under a fixed μ. Since the Newton method may lead to a local optima, we can run the algorithm with different initial values and select the best one.

In addition, we also design a rounding algorithm to obtain a feasible 0–1 solution based on the continuous optima achieved above. In each iteration, the rounding algorithm greedily sets Y_{ij} as 1 to maximize the objective function in Eq. (5) while still satisfying all constraints. To make our integer solution closer to the optimum one, the rounding order of queries matters. We always start with queries with the most keywords first, because chances that they overlap with other queries are higher. Once their grouping result is determined, other queries can be assigned to the corresponding groups.

We consider a dictionary that consists of (w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8) and two sample queries are as shown in Table II(a) and II(b). We also assume $c=c'$. To show how it affects grouping results, the constraint of search delay will be set differently. Table III shows grouping results under different delay constraints using our Mathematic Relaxation.

2) Projected Gradient Descent Method: The Newton method is quite simple and gives a relatively fast rate of convergence. However, this method is very expensive in each iteration - it needs the function evaluation and then the derivative evaluation. If the volume of queries is large, then this might not be a good choice. Thus, to improve the efficiency when faced with large amounts of queries, we consider a simple modification of gradient descent for constrained optimization: a projected gradient descent method. In general, projected gradient algorithms minimize an objective $f(x)$ subject to the constraint that $x\in\chi$ for some convex set χ. They do this by iteratively updating $x:=\prod_\chi(x+\eta\triangledown f(x))$, where η represents a step length of learning rate, and $\prod_\chi=\arg\max_x\{\|z-x\|\,|x\in\chi\}$ is the Euclidean projection onto

TABLE III: Grouping Results using Mathematic Relaxation.

Constraint	Group	Combined Query
4	$G_1 = \{q_1, q_3, q_7\}$	$\hat{q}_1 = 11010000$
	$G_2 = \{q_2, q_4, q_5, q_6\}$	$\hat{q}_2 = 00001111$
3	$G_1 = \{q_1, q_3, q_7\}$	$\hat{q}_1 = 11010000$
	$G_2 = \{q_2, q_5\}$	$\hat{q}_2 = 00001101$
	$G_3 = \{q_4, q_6\}$	$\hat{q}_3 = 00000111$

(a) Example One.

Constraint	Group	Combined Query
5	$G_1 = \{q_1, q_3, q_6, q_7\}$	$\hat{q}_1 = 11010001$
	$G_2 = \{q_2, q_4, q_5\}$	$\hat{q}_2 = 00001111$
4	$G_1 = \{q_1, q_3, q_7\}$	$\hat{q}_1 = 11010000$
	$G_2 = \{q_2, q_4, q_5\}$	$\hat{q}_2 = 00001111$
	$G_3 = \{q_6\}$	$\hat{q}_3 = 10000001$

(b) Example Two.

set χ. First order projected gradient algorithms are effective when second order methods are infeasible because of the dimension of the problem.

3) Greedy Algorithm: Since Mathematic Relaxation uses a first order local optimization method to solve a non-convex optimization problem, of which computational complexity isn't proven, this strategy has no guarantee on time. Thus, we consider a greedy algorithm with a guaranteed bound.

According to section IV-A, combination always brings about cost saving. Here, we reconsider the OQG problem in terms of cost saving. Given a set of queries $Q = \{q_1, q_2, ..., q_n\}$ from n different users over the same database, we group these n queries into k non-overlapping groups $G_1, G_2, ..., G_k$ where the number of keywords in each group is no more than α such that the overall sum of savings $\sum_{i=1}^{k} S(G_i)$, is maximized.

Considering two queries, q and q', we define the saving from merging q with q' as $c' + c(|q| + |q'| - |q \vee q'|)$. In other words, combining two queries, q and q', will save one charge from the owner and overlapping-keyword search cost. Since one token generation cost can be saved by combining any two queries q and q', the more overlapping keywords q and q' have, the more search cost it will save through their combination. If q and q' have a containment relationship, we can prove there always exists some optimal grouping result that contains a group with both q and q', as is shown in Theorem 2.

Theorem 2. *Given a set of queries $Q = \{q_1, q_2, ..., q_n\}$, for any two queries q and q', if the keywords of q are entirely contained in the keywords of q', then there is some optimal grouping strategy that contains a group with both q and q'.*

Proof. Assume the optimal partition P over Q has two groups G_1 and G_2, such that $q \in G_1$ and $q' \in G_2$. Thus, individual group saving of G_1 is $S(G_1) = c'(|G_1| - 1) + c(\sum_{i=1}^{|G_1|} |q_k| - |\hat{q}_1|)$, and the same for G_2. We can obtain the overall savings of G_1 and G_2 :

$$S(G_1) + S(G_2) = \tag{6}$$
$$c'(|G_1| + |G_2| - 2) + c(\sum_{i=1}^{|G_1|+|G_2|} |q_k| - |\hat{q}_1| - |\hat{q}_2|)$$

Moving q to G_2 leads to new groups $G_{1'} = G_1 \setminus \{q\}$ and $G_{2'} = G_2 \cup \{q\}$, with $|G_{1'}| = |G_1| - 1$ and $|G_{2'}| = |G_2| + 1$. q is entirely contained by q', hence $|\hat{q}_{2'}| = |\hat{q}_2|$. Thus, $S(G_{2'}) = c'(|G_2|) + c(\sum_{i=1}^{|G_2|} |q_i| + |q| - |\hat{q}_2|)$. Although it is difficult to directly know the exact value of $|\hat{q}_{1'}|$, we can bound it as $|\hat{q}_1| - |q| \leq |\hat{q}_{1'}| \leq |\hat{q}_1|$, such that $S(G_{1'}) \geq c'(|G_1| - 2) + c(\sum_{i=1}^{|G_1|} |q_i| - |q| - |\hat{q}_1|)$. Listed below is a lower bound of the overall group savings for new groups $G_{1'}$ and $G_{2'}$:

$$S(G_{1'}) + S(G_{2'}) \geq \tag{7}$$
$$c'(|G_1| + |G_2| - 2) + c(\sum_{i=1}^{|G_1|+|G_2|} |q_i| - |\hat{q}_1| - |\hat{q}_2|)$$

Comparing Eq. (6) and Eq. (7), we conclude that, moving q to G_2 yields a new partition, which is at least as good as P and thus still optimal. □

Hence, we can greedily group queries q and q', and treat them cost-wise as a single query q', which can be further merged with other queries. The containment can be easily determined by the OR operation. This is a Naive Greedy solution, of which the time complexity is $O(n^2)$. For the rest of the paper, we assume all such containments have been identified.

It is obvious that, without any constraints on the search delay, the optimal solution is to combine n users as a group and issue a single combined query. However, when constraints are added, it becomes an NP-hard problem, thereby we consider a Greedy Partition solution to efficiently approximate its optimal result with an upper bound. Greedy Partition starts with a single group, iterating to split a group of which the splitting cost is minimal among all existing groups, until all query groups are subject to the search delay constraints. As it is a typical set partition problem, we will consider Problem 1 in terms of set theory as follows.

First, given a query set Q, we define a set function $C : 2^Q \to \mathbb{R}$ where

$$\forall\, G \subseteq Q,\ C(G) = \begin{cases} 0 & G = \emptyset \\ c' + c \cdot |\hat{q}| & \text{otherwise} \end{cases} \tag{8}$$

Then, we can reformulate the OQG problem as below. Given a system (Q, C, k), where Q is a set of queries, $C : 2^Q \to \mathbb{R}$ is a set function, and k is a variable with $1 \leq k \leq n$.

$$\text{minimize} \quad C(G_1) + C(G_2) + \cdots + C(G_k) \tag{9a}$$
$$\text{subject to} \quad G_1 \cup G_2 \cup \cdots \cup G_k = Q \tag{9b}$$
$$G_i \cap G_j = \emptyset \qquad 1 \leq i < j \leq k \tag{9c}$$
$$|\hat{q}_i| \leq \alpha \qquad 1 \leq i \leq k \tag{9d}$$

As is proven in the paper [28], given a nondecreasing submodular system (V, f, k), where $f(V) + f(\emptyset) \geq f(S)$ holds for any nonempty subset S of V, the set partition problem can be approximated within a factor of $(2 - 2/k)$ in polynomial time. This paper provides a greedy algorithm to guard this result. Since our system is also submodular (proven below), we present a Greedy Partition that satisfies delay constraints.

As is shown in Algorithm 1, the Greedy-Partition has two functions. The main function GREEDY-PARTITION returns the final partition P over a given set Q. Starting with the 1-partition $P_1 = Q$, in its ith iteration, we obtain an $i+1$-partition P_{i+1} by partitioning some members of the previous i-partition P_i. We halt when a partition P satisfies the delay constraints. For any member W in i-partition P_i, we call function OPTIMAL-SUBSET [28] to find its minimal-partitioning-cost subset. Since a minimal-cost solution is desired, we

Algorithm 1 Greedy Partition

Input: a system, (Q, C)
Output: a partition over Q, P

function GREEDY-PARTITION(Q, C)
  $P_1 \leftarrow \{Q\}$
  for each $i \in [1, n]$ **do**
    for all $W \in P_i$ **do**
      $(S, W) \leftarrow$OPTIMAL-SUBSET(W)
    $(S_i, W_i) \leftarrow argmin\,(C(S) + C(W/S) - C(W))$
    $P_{i+1} \leftarrow (P_i - \{W_i\}) \cup \{S_i, W_i/S_i\}$
    if each $W \in P_i$ satisfies search delay **then**
      return P_i

TABLE IV: Grouping Results using Greedy Partition.

Constraint	Group	Combined Query
4	$G_1 = \{q_1, q_3, q_7\}$	$\hat{q}_1 = 11010000$
	$G_2 = \{q_2, q_4, q_5, q_6\}$	$\hat{q}_2 = 00001111$
3	$G_1 = \{q_1, q_3, q_7\}$	$\hat{q}_1 = 11010000$
	$G_2 = \{q_2, q_5\}$	$\hat{q}_2 = 00001101$
	$G_3 = \{q_4, q_6\}$	$\hat{q}_3 = 00001111$

(a) Example One.

Constraint	Group	Combined Query
5	$G_1 = \{q_1, q_3, q_7\}$	$\hat{q}_1 = 11010000$
	$G_2 = \{q_2, q_4, q_5, q_6\}$	$\hat{q}_2 = 10001111$
4	$G_1 = \{q_1, q_3, q_7\}$	$\hat{q}_1 = 11010000$
	$G_2 = \{q_2, q_4, q_5\}$	$\hat{q}_2 = 00001111$
	$G_3 = \{q_6\}$	$\hat{q}_3 = 10000001$

(b) Example Two.

choose the least-cost partition among all members. The time complexity of this algorithm is $O(kn^3)$, where k is the number of groups, and n is the query number. The grouping results of the previous examples are listed in Table IV.

In the rest of this section, we will demonstrate that the Greedy Partition can solve the OQG problem within a factor of $(2 - 2/k)$ in polynomial time. According to [28], our proposed algorithm on a given system (Q, C) can achieve the above properties if the following two conditions hold: (1) C is submodular, and (2) C is non-decreasing.

Before showing that Greedy Partition satisfies the above two conditions, we introduce their definitions. Given a finite set V and a set function $f : 2^V \rightarrow \mathbb{R}$, f is (1) submodular if $\forall S \subseteq V$ and $s_1, s_2 \in V \setminus S$, $f(S \cup \{s_1\}) + f(S \cup \{s_2\}) \geq f(S \cup \{s_1, s_2\}) + f(S)$ always holds; (2) non-decreasing if $f(V) + f(\emptyset) \geq f(S)$ holds for any nonempty subset S of V .

Lemma 1. *The set function $C : 2^Q \rightarrow \mathbb{R}$ is submodular.*

Proof. Now, we will show for every $G \subseteq Q$ and $q_1, q_2 \in Q \setminus G$, Eq. (10) always holds. Without loss of generality we can assume that $G \neq \emptyset$, otherwise the answer is immediate.

$$C(G \cup \{q_1\}) + C(G \cup \{q_2\}) \geq C(G \cup \{q_1, q_2\}) + C(G) \quad (10)$$

Since $C(G \cup \{q_1\}) + C(G \cup \{q_2\}) = 2c' + c(|\hat{q} \vee q_1| + |\hat{q} \vee q_2|)$ and $C(G \cup \{q_1, q_2\}) + C(G) = 2c' + c(|\hat{q} \vee q_1 \vee q_2| + |\hat{q}|)$, we need to prove Eq. (11) ≥ 0 always holds.

$$\begin{aligned} &C(G \cup \{q_1\}) + C(G \cup \{q_2\}) - C(G \cup \{q_1, q_2\}) - C(G) \\ &= c(|\hat{q} \vee q_1| + |\hat{q} \vee q_2|) - c(|\hat{q} \vee q_1 \vee q_2| + |\hat{q}|) \\ &= c(|\hat{q} \vee q_1| + |\hat{q} \vee q_2| - |\hat{q} \vee q_1 \vee q_2| - |\hat{q}|) \end{aligned} \quad (11)$$

Assume that, nonnegative integers x, y, z represent the number of overlapping keywords between $\hat{q}$ and q_1, $\hat{q}$ and q_2, q_1 and q_2, respectively. Let m be the overlapping keyword number among $\hat{q}$, q_1, and q_2. It is obvious that $z \geq m \geq 0$.

$$\begin{aligned} &|\hat{q} \vee q_1| + |\hat{q} \vee q_2| = 2\,|\hat{q}| + |q_1| + |q_2| - x - y \\ &|\hat{q} \vee q_1 \vee q_2| + |\hat{q}| = 2\,|\hat{q}| + |q_1| + |q_2| - x - y - z + m \\ &|\hat{q} \vee q_1| + |\hat{q} \vee q_2| - |\hat{q} \vee q_1 \vee q_2| - |\hat{q}| = z - m \geq 0 \end{aligned} \quad (12)$$

Based on Eq. (12), we conclude, for every $G \subseteq Q$ and $q_1, q_2 \in Q \backslash G$, Eq. (10) always holds. Thus, $C : 2^Q \rightarrow \mathbb{R}$ is a submodular set function. □

Lemma 2. *The set function $C : 2^Q \rightarrow \mathbb{R}$ is non-decreasing.*

Proof. Based on Definition 2, we should prove $C : 2^Q \rightarrow \mathbb{R}$, $C(Q) + C(\emptyset) \geq C(G)$ holds for any nonempty subset G of Q. According to Eq. (8), $C(\emptyset) = 0$. Since $\forall G \subseteq Q$, it is obvious that set Q's combined query contains more keywords than its subset G's combined query. Thus, we can obtain $C(Q) - C(G) \geq 0$, hence, $C(Q) + C(\emptyset) \geq C(G)$. □

Theorem 3. *The QOG problem can be approximated within a factor of $(2 - 2/k)$ by Greedy Partition.*

This theorem easily follows from Lemmas 1 and 2. This grouping strategy is suitable for those users who have requirements on cost reductions.

V. FAIR COST SHARING

Our grouping strategies will yield a total cost for n users. Thus, one must find a way to distribute the cost among all users. A major purpose of our proposed grouping strategies is to seek high efficiency of the whole network, in the fields of both finance and computation. As self-interested and autonomous entities, users may behave strategically by misreporting their willingness to query to maximize their profit, thereby harming the efficiency. Thus, we want our cost sharing mechanism to be incentive compatible, i.e., it is in users' best interests to be truth telling [19]. Also, it should provide an incentive for users in their assigned groups to participate in the coalition without coercion, *i.e.*, it is fair and maintains the stability of a given grouping result.

A. Cost Sharing Mechanism

To address this challenge, we design a cost sharing mechanism with two desirable properties: (1) *group-strategyproofness* and (2) *sharing incentive*. In the following, we first present our mechanism, then prove it can satisfy the above two properties. In our cost sharing mechanism, the total cost of n users is composed of two parts: one part goes to the data owner's account, and the other is for Ethereum miners; the same applies for individual cost. Each user is equally responsible for the total payment to the data owner. Given a grouping result of k combined queries, the data owner will make a revenue of kc', each user paying kc'/n to him.

Any keyword in a combined query may be redundant for some of its group members, and it is unfair for a user to pay for

TABLE V: An Example of User Cost Sharing.

Keyword	Cost	Shared by
w_1	$1 \cdot c$	q_1, q_3, q_7
w_2	$1 \cdot c$	q_1, q_3
w_3	$0 \cdot c$	
w_4	$1 \cdot c$	q_1
w_5	$2 \cdot c$	q_2, q_5
w_6	$2 \cdot c$	q_2, q_4, q_5
w_7	$1 \cdot c$	q_4, q_6
w_8	$2 \cdot c$	q_2, q_4, q_6

(a) Cost of each keyword

users	Cost
q_1	$\frac{3}{7}c' + (\frac{1}{3} + \frac{1}{2} + 1) \cdot c$
q_2	$\frac{3}{7}c' + (\frac{2}{2} + \frac{2}{3} + \frac{2}{3}) \cdot c$
q_3	$\frac{3}{7}c' + (\frac{1}{3} + \frac{1}{2}) \cdot c$
q_4	$\frac{3}{7}c' + (\frac{2}{3} + \frac{1}{2} + \frac{2}{3}) \cdot c$
q_5	$\frac{3}{7}c' + (\frac{2}{2} + \frac{2}{3}) \cdot c$
q_6	$\frac{3}{7}c' + (\frac{1}{2} + \frac{2}{3}) \cdot c$
q_7	$\frac{3}{7}c' + \frac{1}{3} \cdot c$

(b) User individual cost

a keyword he never requests. Thus, the total cost of searching a certain keyword is only borne by those users who request it. Thus, the cost sharing is at the granularity of n users instead of each group. For each unique keyword, we calculate its total cost in all combined queries, and then evenly distribute the cost among all users querying this keyword. That is, if a keyword is queried by m of n users and appears in t of k combined queries, its total search cost is tc, and each one from m users is equally responsible for a cost share of tc/m.

We show how to share the total cost using the grouping result shown in Table IV(a) under the constraint of 3 keywords. For the rest of this paragraph, each user i is identified by his query q_i. The grouping result is $G_1 = \{q_1, q_3, q_7\}$, $G_2 = \{q_2, q_5\}$, and $G_3 = \{q_4, q_6\}$. Thus, the cost paid to the corresponding data owner is $3c^{'}$, equally distributed among 7 users. Table Va presents the total cost for each keyword and who should be fairly responsible for the corresponding cost. Table IV(b) gives the final split cost for each user. For example, user 1's total cost is $3c'/7 + (1/3 + 1/2 + 1)c$, where (1) $3c'/7$ is paid to the data owner, shared with all other 6 users; (2) $c/3$ comes from querying keyword w_1, shared with users q_3 and q_7; (3) $c/2$ comes from querying keyword w_2, shared with q_3; (4) c comes from querying keyword w_4 by himself.

B. Theoretical Analysis

We present theoretical analysis to demonstrate that our cost sharing mechanism achieves some desirable properties. For group-strategyproofness, we should demonstrate that each user will honestly disclose his real query request, even if they are permitted to collude. For each keyword, if a user's dominant strategy is to truthfully tell whether he wants to query it or not, then truth-revealing is his dominant strategy. Thus, we can divide the whole proof into d steps, and the j-th step shows that each user would prefer revealing his real request on the keyword w_j in our cost-sharing mechanism. Thereby, we divide our cost sharing mechanism on keyword search part into d cost sharing methods, one for each keyword, then we prove each method satisfies group-strategyproofness.

The cost sharing method of keyword w_j is a function, ξ_j, which distributes the total cost of searching for the j-th keyword, denoted as C_j, to its requesters. More formally, ξ_j takes two arguments, a subset of users G and a user q_i, and returns a nonnegative real number satisfying the following: (1) if $q_i \notin G$ then $\xi_j(G, q_i) = 0$, and (2) $\sum_{q_i \in G} \xi(G, q_i) = C_j$. As is proven in [29], if ξ_j is a cross-monotone, then the mechanism specified above is group-strategyproof- for keyword w_j. Thus, we need to prove ξ_j is cross-monotone. A cost sharing method can be said to be cross-monotone if for $G \subseteq R$, $\xi_j(G, q_i) \geq \xi_j(R, q_i)$ for every $q_i \in G$.

Lemma 3. *For every $j \in [1, d]$, ξ_j is cross-monotone.*

Proof. Any $q_i \in R \setminus G$ refers to a user not requesting the j-th keyword, thereby they are charged zero cost share. Thus, $G \subseteq R$, $\xi_j(G, q_i) = \xi_j(R, q_i)$ for every $q_i \in G$. Thus, ξ_j is a special cross-monotone cost sharing mechanism. □

Theorem 4. *Our cost sharing mechanism satisfies group-strategyproofness and sharing incentive for all users.*

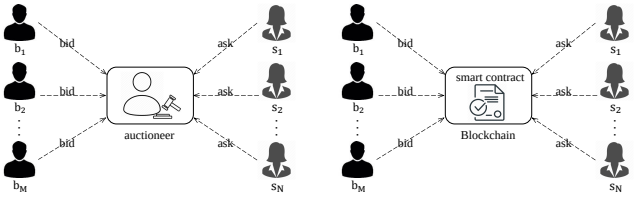
Proof. The property of group-strategyproofness can be proven using Lemma 3. To show sharing incentive, we should reveal that for any user, leaving his current assigned group would not bring him more benefits. Sending an individual query definitely brings more cost paid to the data owner, which is cost-inefficient. As is shown in Eq. (3), grouping is always beneficial for each user to save cost paid to data owners. Thus, no one has incentive to leave. □

VI. Multiple Substitute Data Owners

Previously, we focus on the query grouping problem under the assumption that a specific data owner and the price of his database, *i.e.*, c', are given. In reality, many owners of substitute commodities coexist in a data market for a data buyer to choose from. Given the existence of multiple owners, we could foresee that the competition of low prices harms all owners' interests and makes the data market move towards melting malignancy. Meanwhile, individual owners may have different attitudes to privacy leakage when monetizing their data. A technique called differential privacy allows a data owner to guarantee his data privacy in a self-controllable manner. By applying differential privacy, a noisy version of data is provided. It is an aggregated query answer added with some random noise. The magnitude of random noise directly impacts data privacy loss and can be personalized by the owner herself. Obviously, less privacy loss lowers the data utility from a buyer's perspective, and also, discounts the valuation of the owner's database. Definitely, a buyer should pay less if he gets an inaccurate result. This indicates that data pricing should be inversely proportional to the privacy protection level. Given different privacy protection levels with different costs, a buyer can pair with a suitable seller among alternative owners under his budget constraint and inaccuracy tolerance. However, a selfish owner may cheat for more benefits. Thus, we want it so that all data owners can truthfully report their privacy protection levels to data buyers.

In this section, we aim to design a data trading mechanism to satisfy two objectives in the below: (1) a pricing policy that enforces each data owner to report his privacy protection level truthfully, and (2) a matching policy that pairs buyers and sellers efficiently. To assist the matching and administer the trading conduction between buyers and sellers, a trusted third party is necessary. In particular, a double auction fits well with the bilateral nature of this scenario. Fig. 3(a) shows a traditional double auction model. The trusted third party in

a double auction is the auctioneer between data users (buyers) and data owners (sellers). Buyers and sellers submit bids and asks to the auctioneer, and then he needs to determine (1) the winners among all buyers and sellers, (2) the way of matching the winning buyers and winning sellers, and (3) the price it charges the buyers and the price it rewards the sellers. In the traditional double auction model, the auctioneer gains from price differences.



(a) Traditional double auction. (b) Decentralized double auction.

Fig. 3: Double auction between sellers and buyers.

A. A Decentralized Double Auction Model

Thanks to the application of Ethereum blockchain, we propose a decentralized double auction model which gets rid of third-party intervention and automatically auctions between buyers and sellers. Next, we describe the double auction model in a blockchain-based decentralized data market. Fig. 3(b) displays three logical roles in the proposed auction model, where the trusted auctioneer is replaced by a smart contract.

- Buyers: A set of N buyers aim to query an identical type of database. Each buyer should submit a bid to indicate his bidding price as well as his minimal requirement on the result accuracy. Here, we identify the i-th buyer based on his bidding vector $b_i = [bid_i,\ r_i]$, where bid_i is his biding price and r_i is his accuracy requirement. The biding price bid_i should reveal the buyer's valuation about the required database. Note, a buyer here can be considered a representative or a proxy of those buyers from the same group. We assume that all those grouped buyers reach agreements on the bidding price as well as the result accuracy requirement, and act as a single buyer in regards to the outside.
- Sellers: A set of M sellers are able to answer all buyers' queries with substitute databases. Each seller also needs to submit an ask to indicate his asking price as well as his requirement on the privacy loss. Since the privacy loss is inversely proportional to the query answer quality, we use the answer quality a seller can provide to reflect his privacy protection requirement. Here, we identify the j-th seller based on his asking vector $s_j = [ask_j,\ a_j]$, where ask_i is his asking price and a_j is his accuracy quality. Each seller should truthfully report the accuracy quality he can provide.
- Smart Contract: Given B and S, the one-round execution of this smart contract should give the results of the winning buyer set $B_w \in B$, the winning seller set $S_w \in S$, the matching between B_w and S_w, the price P_i^b that the winning buyer b_i is charged, and the payment P_j^s that the winning seller s_j is rewarded. In fact, $P_i^b \geq P_j^s$ always holds for any matched buyer-seller pair (b_i, s_j). Since all miners will execute this smart contract, decisions can be obtained in a decentralized and trust-worthy manner. The difference between the total charges and the total payments will be regarded as transaction fees for the corresponding miner.

B. Utility of Buyers and Sellers

Given a buyer-seller mapping, $i = m(j)$, which ensures that $a_j \geq r_i$, the utility of buyer b_i and that of seller s_j are respectively defined as follows:

$$U_i^b = \begin{cases} a_j \cdot bid_i - P_i^b & b_i \in B_w, \\ 0 & otherwise. \end{cases} \tag{13}$$

$$U_i^s = \begin{cases} P_j^s - ask_j & s_i \in S_w, \\ 0 & otherwise. \end{cases} \tag{14}$$

Accordingly, we can obtain the utility on the buyer side, denoting $U^B = \sum U_i^b$, and the utility on the seller side, denoting $U^S = \sum U_i^s$, respectively.

Definition 1. *(Social Welfare). The social welfare for all buyers and sellers is defined as*

$$U_{sw} = U^B + U^S = \sum\nolimits_{j \in S_w} a_j \cdot bid_{m(i)} - ask_j. \tag{15}$$

C. Objective and Desirable Properties

The objective of this work is to design strategy-proof auction mechanisms for the data market that maximizes the overall social welfare, while satisfying buyers' result quality requirement.

$$\text{maximize} \quad U_{sw} \tag{16a}$$

$$\text{subject to} \quad r_i \leq a_j \quad \text{if} \quad i = m(j) \tag{16b}$$

$$P_i^b = P_j^s \quad \text{if} \quad i = m(j) \tag{16c}$$

To understand the meaning of strategy-proof in the setting of auctions, we firstly introduce the concept of truthfulness and individual rationality. Above these two, we also introduce computational efficiency as another design target.

- Truthfulness: An auction mechanism is truthful if playing (bidding or asking) truthfully is a weakly dominant strategy for each player (buyer or seller) that only concerns about his/her own utility. In other words, no buyer can improve his utility by submitting a bid different from his true valuation, and no seller can improve his utility by submitting an ask different from his true cost. Specifically, it implies the following for our auction model: each buyer should report his true valuation on the query result, and each seller should report the true answer quality he can provide.
- Individual rationality: No winning buyer is charged more than his bid, and no winning seller is rewarded less than his ask. With respect to our auction model, this means that for every winning matching between $b_i \in B_w$ and $s_j \in S_w$, $P_i^b \leq bid_i$ and $P_j^s \geq ask_i$ holds.
- Computational efficiency: The auction outcome, which includes the winning sets of buyers and sellers, their mapping, and the clearing price and payment, is tractable with a polynomial time complexity.

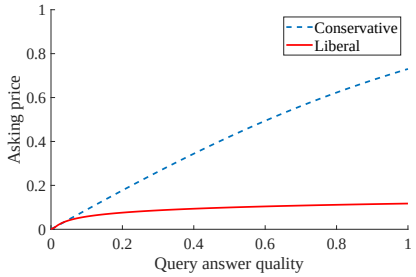


Fig. 4: Pricing private data given $V = 1$.

D. Valuating Private Data

As we mentioned before, we want it so that each seller truthfully reports his privacy protection level, *i.e.*, his answer quality. Thus, we build a weak binding between the asking price and the answer quality when a seller submits his ask. When reporting a certain value of his answer quality, the seller's asking price cannot exceed an upper bound, which is predefined in the smart contract. Otherwise, his asking will be discarded as invalid. In the following, we provide a framework to figure out different upper bounds for different answer qualities. Given the guidance on the upper bounds, each seller valuates his database based on his privacy loss and risk attitude. The risk attitude here describes how a seller could lower his price in order to get more buyers.

For simplicity, we just assume that all substitute databases have an equal value V. Then a seller discounts the value of his database with the answer quality q to V_q. Thus, his asking price should never exceed that value, *i.e.*, $asking \leq V_q$. Since different sellers have different risk attitudes, they can determine their final asking prices based on their individual risk attitudes. We divide sellers into two types: conservative and liberal. A conservative seller will apply a logarithm function, shown in Eq. (17a), to determine his final asking price, while a liberal seller will choose a sublinear function, shown in Eq. (17b). Fig. 4 gives an example under the condition of $V = 1$.

$$ask = \frac{22Vq}{\sqrt{500 + 250q^2}} \tag{17a}$$

$$ask = \frac{V \log_2(9000q^2 + 1)}{112} \tag{17b}$$

E. Truthful Auction Design

To maximize the social welfare as shown in Eq. (15), it is natural to select sellers with high qualities and low costs to match the buyer with higher bidding prices. Obviously, those liberal high-quality sellers would be favored. In this section, we present a design to get a unique solution for our proposed auction model. It contains two sub-procedures specified in Algorithm 2 and Algorithm 3, which correspond to two stages, one is for winner determination and the other is for pricing, respectively.

Firstly, we sort the sellers in non-decreasing order by the ratio of asking prices and answer qualities, and sort the buyers in non-increasing order by their bids. After sorting, the winner determination process greedily matches the buyers and the sellers. That is, a buyer always chooses a seller with the highest cost efficiency while still satisfying his result accuracy requirement. As a database is a digital good, a seller can infinitely sell it. Thus, buyers can choose the same seller. Algorithm 2 shows the pseudo-code of the winner selecting algorithm. We can see that the running time of Algorithm 1 is $O(NM)$, which is polynomial time.

Algorithm 2 Winner Determination

Input: Set of buyers B, vector of query result quality requirement r and bidding price bid, set of sellers S, vector of query result quality guarantee q and asking price ask.
Output: Set of winning buyers B_w and winning sellers S_w, buyer-seller mapping matrix m.

function WINNER-DETERMINATION(B, S)
  $B_w \leftarrow \emptyset$, $S_w \leftarrow \emptyset$, $m_{N\times 2}$
  Sort S by the ratio of asking prices and answer qualities in non-decreasing order: $\frac{ask_1}{q_1} \leq \frac{ask_2}{q_2} \leq \cdots \leq \frac{ask_M}{q_M}$
  Sort B by bids in non-increasing order: $bid_1 \geq bid_2 \geq \cdots \geq bid_N$
  for all $i \in [1, N]$ **do**
    for all $j \in [1, M]$ **do**
      if $bid_i \geq ask_j$ and $r_i \leq q_j$ **then**
        Add $[i, j]$ to m
        $B_w \leftarrow B_w \cup i$
        $S_w \leftarrow S_w \cup j$
        break
    Add $[i, -1]$ to m
  return B_w, S_w, m

Algorithm 3 VCG Pricing

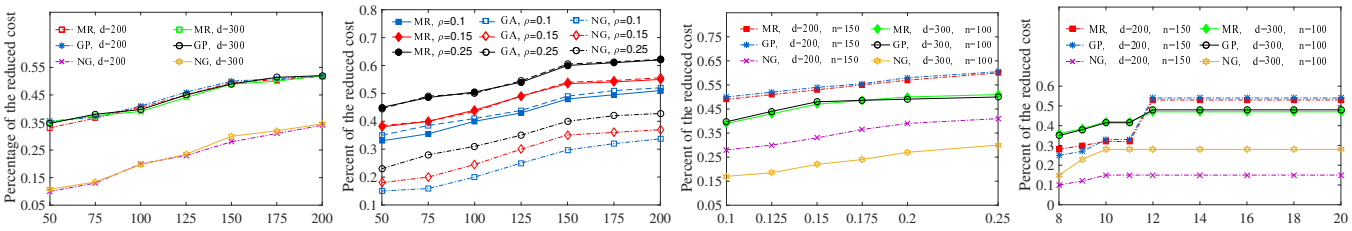
Input: Set of winning buyers B_w, set of winning sellers S_w, and buyer-seller mapping matrix m.
Output: Vector P^b of charges for buyers B_w and payments P^s to sellers S_w.

function WINNER-DETERMINATION(B, S)
  $P^b \leftarrow [0]_N$, $P^s \leftarrow [0]_M$
  for all $i \in [1, N]$ **do**
    if $b_i \in B_w$ **then**
      $P^b_i = bid_{mf}$
  for all $j \in [1, M]$ **do**
    if $s_j \in S_w$ **then**
      $P^s_j = ask_{mf}$
  return P^b, P^s

In the pricing phase, we apply VCG mechanism to obtain truthfulness, which is shown in Algorithm 3. Each winning buyer pays the highest failure bidding price, and each winning seller receives the highest failure asking price. It is obvious that the time complexity of Algorithm 3 is $O(N + M)$.

VII. Performance Evaluation

Our evaluation consists of two parts. In the first part, we focus on evaluating our proposed cooperative search scheme on real query traces AOL [30]. In the second part, we implement an Ethereum testbed to demonstrate the practicality of



(a) X-axis : number of users. (b) X-axis : number of users. (c) X-axis : keyword density. (d) X-axis : delay constraint.

Fig. 5: Evaluations of grouping strategies on real query traces. MR: Mathematic Relaxation, GP: Greedy Partition, and NA: Naive Greedy.

our scheme, and also analyze the actual relationship between the keyword number and the search delay.

A. Cooperative Search Scheme

Our experiments evaluate two grouping strategies in terms of total cost reductions and the cost sharing mechanism in terms of individual cost saving. Mathematic relaxation was implemented with MATLAB-R2017b and the greedy algorithms were implemented with Eclipse 4.6 in Java. All experiments are conducted on AOL. As AOL is a huge query collection, we randomly choose 200 users with 31 804 queried keywords in total, among which 17 786 are unique. Thus, a 400×17786 binary matrix is constructed to reveal each query's request on each keyword. Since it is still a large array, we semi-randomly select part of the matrix in each experiment to satisfy pre-set constraints on dictionary size, query number, and keyword density. For simplicity, we define two parameters: *keyword density* and *charge ratio*. Given a d-size dictionary and an n-query set Q, *keyword density* ρ of Q is defined as $\rho = \sum_{i=1}^{n} |q_i| / (n \times d)$. Given c' from a data owner and c from miners, *charge ratio* r is defined as $r = c'/c$.

Grouping strategies: We analyze the percentage of reduced total cost using our proposed grouping strategies: Mathematic Relaxation (using PGD here since the query volume is large), Naive Greedy and Greedy Partition. Fig. 5 shows, in all parameter settings, all strategies achieve cost reduction by at least 24.8%. Greedy Partition works slightly better than Mathematic Relaxation, and Naive Greedy achieves the least total cost reduction, which is around 50% of the other two strategies. Since the complexity of Naive Greedy and Greedy Partition is $O(n^2)$ and $O(n^3)$, respectively, we could see an inevitable tradeoff between efficiency and performance. Now, we analyze how each parameter influences the total cost reduction. In Fig. 5(a), as n increases, the total cost reduction also increases. Given a fixed ρ, changing d has little effect on the cost reduction. Fig. 5(b) reflects, as ρ increases from 0.1 to 0.25, the total cost is reduced by about 10% for each unique n. In Fig. 5(c), we have two set comparable parameters: $(d = 200, n = 150)$ and $(d = 300, n = 100)$. Given a fixed ρ, each set has the same number of 1s. From this experiment, we could see the first set has more cost savings, since a smaller size of d yields higher chances of keyword overlapping. Fig. 5(d) reflects that the effect of delay constraint α on the total cost reduction decreases as its value increases.

Cost sharing mechanism: In the second part, we study individual cost saving under our cost sharing mechanism by picking up 10 users with $d = 100$. Each time, we change r and select a better one from grouping results from Mathematic Relaxation and Greedy Partition. We compare the cost reduction between individuals and the average. As is shown in Fig. 6, individuals can benefit from our grouping strategies. Besides, no user largely deviates from the average level, which shows our cost sharing mechanism can achieve fairness.

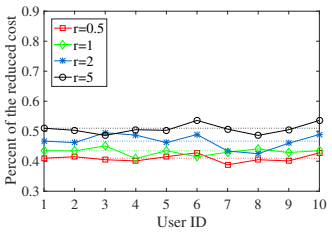


Fig. 6: Individual vs average saving.

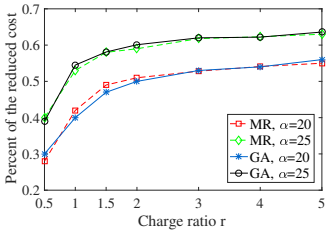


Fig. 7: Effect caused by charge ratio.

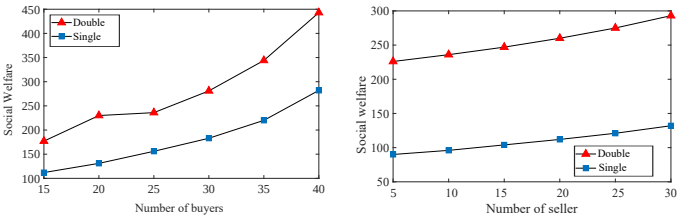
Summary: Both grouping strategies are local optimal. Mathematic Relaxation uses random restarts to produce multiple rounds to mitigate this problem. The larger the number of random restarts, the better the performance, but the longer the execution time. Therefore, Greedy Partition is more appropriate for large scale query systems, and Mathematic Relaxation can be used as the baseline to measure the grouping quality. In terms of the cost sharing mechanism, each user can achieve cost savings near around the average saving.

B. Double Auction

To show the efficiency of our proposed double auction mechanism, we compare the social welfare yielded by the double auction with that yielded by a single auction in the buyer side. We consider two different settings. In the first setting, we assume the seller set is fixed, and we increase the number of buyers. As is shown in Fig. 9(a), the double auction always brings a higher social welfare to the whole data market. In the second setting, we fix the buyer set and allow more and more sellers to join in the auction. The result is given in Fig. 9(b), which is quite similar with what we have in the first setting.

C. Ethereum Testbed

To demonstrate the practicality of our scheme, we implemented a testbed in a simulated Ethereum network called TestRPC [31]. TestRPC is a fast and customizable blockchain emulator. It sets mining time as 0 while truly revealing execution time and gas consumption of a transaction. This design allows us to focus on the search delay itself without being affected by mining or waiting delays. Our Ethereum testbed



(a) Number of seller is fixed as 50. (b) Number of seller is fixed as 50.

Fig. 8: Decentralized double auction between buyers and sellers.

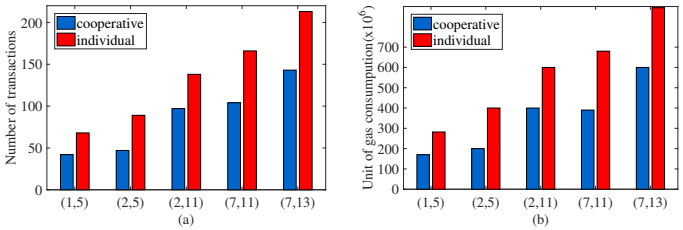


Fig. 9: Cost reduction using testbed (X-axis : number of (overlapping files, unique files) tuples in non-grouping search result).

can be helpful in revealing the real relationship between the number of keywords and search delay by the miners. This provides a better estimate for search delay in the real system, and thereby, a better estimation of the grouping constraint.

Keyword number and search delay: We conduct two experiments to verify the actual relation between the keyword number and search delay. We stored a 5KB database with 20 keywords and 30 files, each tagged with one or more keywords. In the first experiment, we randomly select 5 keywords and incrementally add 20 more each time to see how the execution time changes. The result shows that, as the number of keywords increases, the delay time also increases, though there exists a slowdown in its growth rate. In the second experiment, we dedicatedly design 5 query sets, each including 4 queries. The total keyword number in each set is fixed at 10, while the unique keyword number changes. For each query set, we execute them in two ways: (1) executing all queries individually, and (2) executing a single query composed of 4 queries. We compare accumulative execution times and combined execution times, and analyze how execution time reduces as the number of overlapping keywords increases. The result of this experiment shows that the relationship between the number of overlapping keywords and execution time is nearly proportional. Based on the above results, we conclude that the search delay is at least sublinear to keyword numbers.

Charge ratio: In our real implementation, we store a 1.4MB database with 300 unique keywords and 2000 files. Each file is tagged with some different keywords. We issue 75 transactions in order to store the entire database in blockchain. When previously evaluating our cost sharing mechanism, we find that charge ratio r can affect individual cost savings as well as the total cost reductions. Thus, when performing experiments on our testbed, we first analyze how charge ratio r can affect grouping results, hence changing cost reductions. As is shown in Fig. 7, there is a positive sublinear relationship between the total cost reduction and charge ratio r. In our previous sections, assuming $r = 1$ to yield a maximal reduction on total cost is acceptable, since it can be adjusted by a factor.

Four-user cooperative search: We also envision a small four-user setting with different queries, and conduct several optimal cooperative searches and their individual searches. Fig. 9 reflects the cost reduction in the form of the transaction number and the gas consumption amount, both of which are important cost measures in Ethereum. These two parameters follow a very similar changing pattern if given the same inputs. The reason for this is that each transaction invokes the execution of the same search function. As we can see, the cost reduction is positively related to the ratio of the overlapping matched file number and the unique matched file number, which is a reflection of the overlapping keyword number in original queries.

Summary: Using our testbed, we analyze the actual relation between the keyword number and the search delay, which is sublinear. Experiments are conducted to see how charge ratio affects cost reduction. The pricing for the search part has more effects on the total cost reduction compared with the owner's pricing. The last experiment on the four-user cooperative search has demonstrated the practicality of our proposed scheme.

Discussion: In terms of delay time, as we mentioned in section VII-C, an Ethereum-based data market has extra transaction-waiting time and mining delays compared with a traditional centralized model. Currently, a transaction on the Ethereum blockchain has a pending time between 30 seconds and 16 minutes, which is acceptable. A longer delay is possible due to the transaction volume of Ethereum platform. The reason why we choose Ethereum is just because we do need a platform that enables blockchain as well as smart contract. We can either switch to other less popular platforms, such as CITA, or we can implement our own platform, where a permissioned blockchain is applied, to speed up the transaction processing.

VIII. Conclusion

In this paper, we present a cooperative search scheme on an Ethereum-based data market. We take advantage of smart contract and gas system in Ethereum to separate a query cost into two parts: one for data owners and the other for miners. We also make use of grouping strategies to provide efficiency and cost savings for the users. We provide three methods, suitable for different scenarios, to compute an efficient grouping result. Additionally, we propose a fair cost sharing mechanism to split the total cost among users given a grouping result. This mechanism guarantees some desirable properties such as group-strategyproofness and sharing incentive to avoid free-riders. The experiment results show that our scheme is efficient in terms of cost reduction for both the group as a whole and the individuals.

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